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OPTIMIZING THE TRADEOFF BETWEEN ACCURACY AND CARTOGRAPHIC COMMUNICATION FOR CHOROPLETH MAPS

by

(C) KIRSTY JANET BURT

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled 'Optimizing the Tradeoff between Accuracy and Cartographic Communication for Choropleth Maps' submitted by Kirsty Janet Burt in partial fulfilment of the requirements for the degree of Master of Arts.

Dedication

To my parents, with love.

Abstract

Choropleth maps are constructed to display intangible ratio distributions, such as population density, using data which are confined to administrative units. The density values are represented visually by a range of grey tones. The controversies associated with choropleth map design have evolved with technical and theoretical developments in thematic cartography, and are characterized by a persistently recurring emphasis on the loss of accuracy inherent in the generalization of data for mapping purposes. Traditionally, the data to be presented on a choropleth map were highly generalized; a considerable loss of accuracy was accepted as a reasonable tradeoff for the perceived advantages of simplicity, for both cartographer and map viewer. Recent advances in computer–assisted cartography prompted the suggestion and development of statistically accurate, high quality unquantized choropleth maps, possessing a different shade of grey for each unit on the map. A resurgence in the debate over generalization in choropleth mapping ensued, centering on arguments for and against the unquantized map. This debate lacked definite issues and framework. The central aim of the project became an approach to definition and solution of the quantization controversy in choropleth mapping.

The guiding framework for this research was based on the currently accepted paradigm of cartographic communication in thematic cartography. Successful cartographic communication is defined as the transmission of meaningful information to the viewer via the map. Using this framework, the key problem in choropleth mapping was defined as the controversy over selection of a threshold generalization level which optimizes the tradeoff between accuracy and cartographic communication.

A hypothesis was formulated as the initial step in approaching the defined problem; that an optimum generalization level exists for choropleth maps. Viewer responses to a series of test maps, with a range of generalization levels from highly generalized to unquantized, were collected in the form of three class generalizations of each test map. Variations in viewer response maps were analyzed using tests that were designed to incorporate the two dimensional nature of response arrays. Results of analysis, combined with findings obtained through experience of the research, provided the basis for the major conclusions of this research.

The study results corroborated those of previous research in choropleth map perception, establishing that map viewers can derive meaning from choropleth maps with a much higher number of classes than traditionally anticipated. Analysis indicated that a clear optimum generalization level of fifty classes existed for this data set. These observations formed a basis for the conclusion that the quantization controversy in choropleth mapping is no longer necessary, and for reformulation of the original hypothesis as follows: that an optimum generalization exists for each data set and mapping purpose in choropleth mapping.

The results and observations in the research provide evidence that many complex factors influence the ways in which generalization may affect the visual appearance of information to the map viewer. Selection of an optimum generalization level should depend upon a careful analysis of these factors. Based on this premise, a set of practical guidelines to be applied in the selection of an optimum generalization were formulated.

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I. Developments and Issues in Thematic Cartography

A. Introduction

Rapid changes in the discipline of cartography over the past twenty years have created a research atmosphere which has resurrected an interest in the study of quantization levels on choropleth maps. In 1973, Tobler suggested the use of unquantized choropleth maps, which portrayed information with complete accuracy. These accurate, but visually complex, maps were in direct contrast to the simplified, inaccurate choropleth maps which were generally accepted at that time. Originally, proponents of the broadly generalized choropleth map rejected Tobler's suggestion on both theoretical and practical grounds. Theoretical arguments were based on studies conducted in the early sixties, which established that people could only distinguish up to eleven grey levels on choropleth maps(Jenks and Knos, 1961). Practical arguments originating from the poor visual quality of plotter-produced hatching patterns, were refuted by the development of unquantized maps with high quality design standards(Muller and Honsaker, 1978). Thus, the remaining issues concerning quantization levels on choropleth maps were chiefly theoretical. Results of research into unquantized choropleth maps(Muller, 1978; Peterson, 1979) have indicated that people can extract meaningful information from unquantized maps. It has become apparent through the confusion and disagreement created by the developments and research in unquantized mapping that the central issue in choropleth mapping must be viewed in a more specific sense than the simple dichotomy between quantized and unquantized maps. The central issue has been refined and identified as the establishment of an optimum quantization level between the extremes representing either accuracy or simplicity in choropleth mapping. It is the objective of this thesis to examine this issue in the specific area of choropleth mapping, and also in the more general area of research in cartographic communication.

This literature review will provide a brief outline of the development of changes in thematic cartography, and their effects on cartographic research. This outline will act as a useful historical and practical framework within which to examine a detailed formulation of this thesis topic, and also as a means of placing the topic within the context of theoretical trends in cartographic research.

B. Major Factors Affecting Thematic Cartography

Two major factors affecting thematic cartography over the past twenty years have been identified. These factors are: the importance and identification of the field of thematic cartography, and an increase in capabilities within the technical aspects of thematic cartography(Morrison, 1974).

Identification and Importance of Thematic Cartography

There has been a considerable increase in the importance and scope of maps in modern life(Kolacny, 1978). The increase is chiefly a result of growing demands from areas such as education, advertising, and planning, which make extensive use of thematic maps. As special purpose thematic mapping has developed, it has become necessary to focus on the depiction of highly abstract, non-visual phenomena and relationships, such as social and demographic patterns, with the representation of subtle distinctions(Robinson and Petchenik, 1976). The difficulties encountered in attempts to represent non-visual patterns in visual form via the map have stimulated an interest in theoretical concerns among today's cartographers, in areas such as map design, symbolism, mapping purposes, and many others.

The concern with a theoretical study of cartography is exemplified by the recent actions of cartographers toward establishing an identity for cartography among the scientific disciplines. These actions include: the founding of professional societies of cartographers, the publication of cartographic journals, establishment of cartography courses and departments at the university level, and active research in thematic cartography(Morrison, 1974). The strong research impetus in thematic cartography has fostered the initiation of a theoretical framework for the discipline of cartography, a development which must occur if a valid identity for cartography is to be established.

Impact of Computer Technology on Cartography

Computer technology has permeated all areas of cartography over the past twenty years. The growing impact of computer technology has necessitated the definition of computer-assisted cartography. This is a general term, applying to all aspects of cartography where the computer is used for automation or as an analytical aid(Taylor, 1980).

The growth of computer-assisted cartography has been an accelerating force in the emergence of a cartographic identity(Morrison,1980). The effects of computer-assisted cartography are apparent in several areas of thematic cartography. Automated mapping has freed thematic cartographers from many manual drafting tasks, allowing them to concentrate on methods, map design, and research. The increased number of options afforded by computer mapping allows the cartographer to select a "defendable design"(Morrison,1980), or the best, rather than necessarily simplest, technique to portray his intended message. Finally, the high degree of precision of methodology required in applying computer-assisted cartography has produced specific definitions of current cartographic methodology and terms(Morrison,1980). This high level of precision is contributing to a more rigorous description of the cartographic discipline itself.

The emphasis on thematic cartography, and the rise of computer-assisted cartography have been the chief factors influencing cartography over the past twenty years. The combined force of these factors is exhibited by changing trends in the issues and areas of research currently explored by cartographers.

C. Cartography as a Communication Science

Cartographers have identified an urgent need for a general theory of cartography(Kolacny,1978; Morrison,1978; Ratajski,1978; Rhind,1980; Robinson and Petchenik,1976). It has been suggested that this theory would provide a basic structure for the discipline of cartography, by giving relevance and location to research done in the pursuit of this structure, and by making clear those areas which required further investigation(Robinson and Petchenik,1976). Morrison(1978) acknowledges that in order to establish a unified body of theory, scientists must agree on a fundamental paradigm. The fragmented research activity in cartography has not yet resulted in a broad research paradigm specific to the field of cartography(Robinson and Petchenik,1976). However, cartographers have attempted to define the unique nature of cartography, and use this definition as a guide to examining the theoretical structure of established sciences for possible applications to the development of cartographic theory.

Until recently, the emphasis in cartography has been on the creation and production of the map itself. However, as a result of the growing importance of thematic cartography, combined with the current rise in the use of computer-assisted cartography, emphasis has shifted heavily to an interest in the map viewer. The map has been recognized as a communication device which allows the transmission of spatial information between the cartographer and the map viewer(Kolacny, 1978; Ratajski, 1978; Robinson and Petchenik, 1976). Establishment of the essential processes of cartographic communication by Kolacny(1978) and Ratajski(1978), and the growing acceptance of cartography as a communication science as the fundamental paradigm for development of cartographic theory(Kolacny, 1978; Morrison, 1978; Ratajski, 1978) have provided a general, unifying base for cartographic research.

Initially, research in the area of developing cartographic communication theory has been concentrated in the examination of theory of other communication sciences for possible contributions to the communication science of cartography. The two main bodies of theory investigated by cartographers are the scientific information theory of electronic communication, and the theory of cognitive psychology.

Examination and Rejection of Information Theory

One measure of the communication efficiency of the cartographic process is related to the amount of information transmitted between cartographer and viewer via the map(Robinson and Petchenik, 1976). It was the perceived potential to measure this information transfer which attracted cartographers to the information theory developed in the 1940's. Information theory has been a useful guide in diagraming and describing the cartographic communication process. However, information theory has been ineffective as a tool for the explanation of the cartographic communication process in thematic mapping, because it fails to direct attention to the qualitative content of the information presented on a map, thereby ignoring a key purpose of thematic mapping; the transmission of meaningful information as a key to understanding(Guelke, 1978; Meine, 1978; Salichtchnev, 1978).

Investigation of the Theory of Cognitive Psychology

With the objective of augmenting their understanding of the complex relationships between cartographer and map viewer, cartographers have begun to shift

their exploration of the theory of other communication sciences from information theory to cognitive psychology, a field in which communication theory is derived from a study of the mental processes that receive, transmit, and operate on information(Moates and Schumacher, 1980). The attempt at understanding the cognitive processes of the map viewer, and speculation on the potential effects of this understanding on map construction and cartographic research have become the chief areas of interest in the expansion of cartographic theory.

In cognitive psychology, communication is viewed as the process wherein thought originating in one human mind is converted by that mind into physical forms, and meaning is constructed in the mind of the receiver in response to the physical stimulus. The physical means of communication do not carry meaning; they trigger or release it in the mind of the receiver(Petchenik, 1978). Thus, the communication success of a map is measured by the degree of correspondence between the meaning of the map as defined by the intentions of the cartographer, and the meaning created by the viewer in response to the map. In order to achieve the goal of successful cartographic communication, cartographers must examine all relevant characteristics of the specific map audience to which a particular map is directed, and continue research into the basic cognitive processes which people utilize in constructing meaning from maps.

Perception, a term which has been used in cartography without adequate definition or restraint(Petchenik, 1978), is defined in cognitive psychology as the process by which we determine the meaning of the physical stimuli that impinge upon us(Moates and Schumacher, 1980). There are basic features of perception which should be considered in continued research into the cognitive aspects of cartographic communication. These features are outlined by Moates and Schumacher in *An Introduction to Cognitive Psychology*. The first feature of perception which is pertinent to the study of cartographic communication is the active nature of cognitive processes. Closely related to this feature is the goal-oriented character of many cognitive functions. The cognitive processes involved in achieving these goals are controlled, and then terminated when the goals are achieved. The final aspect of perception which may be considered in cartographic communication is the schema. The schema are units which represent a person's internalized knowledge of the world. The schema may direct the

active search for information, influence their interpretation of the information, or be modified by the information it receives. When constructing a map, the cartographer considers the goal of successful communication of spatial information to a potential map audience. If he considers the characteristics and information goals of the potential audience, there will be a greater ensurance that a map is the best possible device for successful communication of the information, and that the information communicated via the map will be meaningful to that audience.

Conclusion

No complete theory or set of principles for cartography has emerged from research into information theory or cognitive psychology, although findings have been applied in specialized areas of cartography, and in outlining the essential concerns and relationships of the communication process in thematic cartography. Most importantly, these findings have acted as a catalyst for cartographic thought; while exploring the applicability of other theories of communication to cartography, cartographers have begun to identify the important features which define the nature of cartography, thereby establishing an increasingly precise base for developing cartographic communication theory. In this thesis, a combination of information theory as it pertains to cartographic generalization, and of the essential features of cognitive psychology as they relate to cartographic communication, will be used as a research guide in an attempt to resolve a key issue in choropleth mapping, and to provide findings which will contribute to development of theory in the area of cartographic communication.

II. Evolution of the Quantization Controversy in Choropleth Mapping

A. Background to Issues in Choropleth Mapping

Introduction to the Choropleth Map

Since choropleth maps were formally defined by J.K. Wright in 1944, they have been a growing source of interest and controversy for thematic cartographers. The choropleth map exhibits intangible ratio distributions, such as population density, as discontinuous data confined to administrative areas. A choropleth map exhibiting apartment density in Toronto as a percentage of occupied dwellings by enumeration unit is displayed in Figure II.1. It is not surprising, therefore, that these maps have gained recent popularity, with the large volumes of census information made available through agencies such as Statistics Canada. The nature of these distributions can be perceived through a rational process, with the aid of a data model(Jenks,1967). The model consists of stepped prisms, which take their shape from the administrative subdivisions, and their height from the statistical information being mapped, as seen in Figure II.2. The varying heights of the prisms often obscure subdivisions toward the back of the model, and a complete view of the distribution is not achieved. The data model is a useful tool for the conceptualization of the general pattern of intangible distributions, but not for their actual representation. The choropleth map is a planimetric substitute for the data model, in which volumetric statistics are represented by shadings that are assigned values by the cartographer.

In constructing a choropleth map, the cartographer must consider the levels and quality of generalization that the final product will reflect. This involves three decisions: selection of the number of classes to be used, the method by which the data will be subdivided into these classes, and the means by which the choroplethic pattern will be reproduced for graphic presentation. The generalization of the original data, produced by these decisions, reduces the statistical accuracy of the resulting map. The perceived tradeoff between statistical accuracy and successful cartographic communication has been the central focus of controversy in choropleth mapping. The controversy has been evolutionary, responding to the technological and theoretical changes in thematic cartography over the past twenty years.

Figure II.1

Figure II.2

TEN CLASS TEST MAP: DATA MODEL

Early Issues in Choropleth Mapping

Conflicts in choropleth mapping surround the decisions of the cartographer, and the ways in which the subjectivity of these decisions affects the communication of information in this map form. The original major issue in choropleth mapping concerned the number of classes into which the original data were to be subdivided for representation on the choropleth map. The field of behavioural psychology, which was influenced by scientific information theory, provided the initial framework within which decisions on the number of classes for a particular map were made. It was argued that man possesses a small, finite capacity, referred to as channel capacity, for making unidimensional judgements(Miller, 1956). The amount of information transferred increased with the addition of information input categories, but levelled off when the channel capacity of approximately seven categories was reached. Jenks and Knos(1961) studied responses to different class levels on choropleth maps, and established that map viewers were unable to differentiate between more than eleven grey tones on a choropleth map. Despite this discovery, the accepted maximum number of grey tones on choropleth maps hovered conveniently around seven. The system was simple for the map reader, who could easily see the patterns set out by the cartographer, and for the cartographer, who could easily construct six or seven shadings. It was realized that generalization on choropleth maps produced error, and a rigorous campaign to discover methods of reducing this error began.

Research into error reduction focused upon the ways in which class limits were constructed. It was felt that the selection of class intervals was the most important step in constructing a choropleth map, since wisely chosen intervals would allow the viewer to gain a clearer understanding of the areal relationships than he could have obtained from the original data, whereas poorly selected intervals would provide only distorted or inaccurate impressions of the distribution(Jenks and Coulson, 1963). Despite the general agreement among cartographers regarding the importance of selecting class intervals, research in this area has remained an anarchic branch of cartography(Evans, 1976). Studies produced countless methods of data classification, leading to the accepted belief that only the specific purpose of the individual map could be used to determine a classification's relative merits(Monmonier, 1977). This acceptance, combined with the

theoretical and technological changes in thematic cartography over the past twenty years, has prompted a renewed interest in the selection of class intervals in choropleth mapping, and created interesting new issues concerning a traditional mapping form.

B. A Framework of Current Research

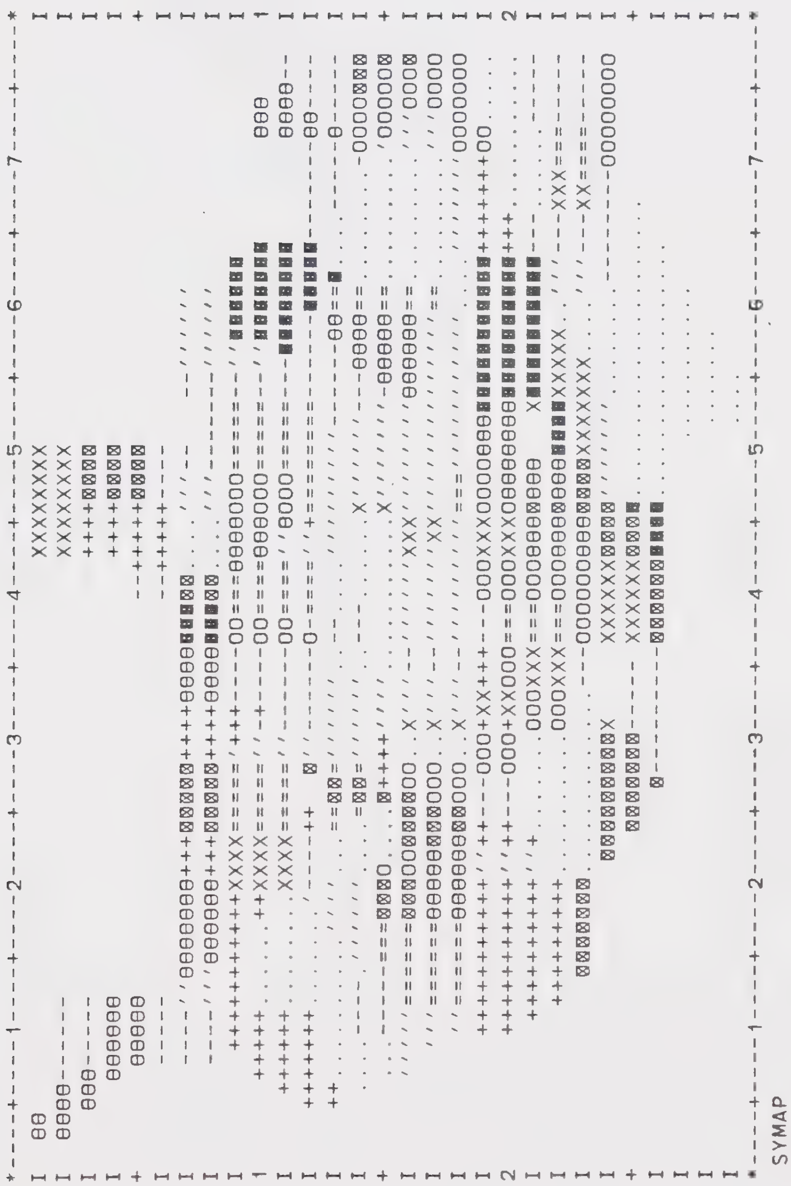
The Role of Computer-assisted Cartography

The changes in choropleth map production, made possible by advances in computer–assisted cartography, have been marked. Traditionally, there were physical, as well as theoretical, reasons for limiting the number of classes on a choropleth map at five to eight. It is now technically feasible to produce virtually continuous shades of grey with various types of automated mapping equipment. The outstanding problem of choropleth maps produced on most automated devices is the coarse texture of the grey tones produced by the line patterns used to achieve the effect of varying density.(see Figure II.3) The facsimile technique developed for choropleth mapping(Muller and Honsaker, 1978) has eliminated the visual static created by line patterns.(see Figure II.1) Facsimile produces a choropleth map on which up to 255 grey tones can be represented, at constant resolution. The degree of detail to be represented on the facsimile choropleth map is the choice of the cartographer; the decision to employ a large number of classes is no longer subject to the poor visual quality of the grey tone pattern produced by most automated methods.

Recent Issues and Studies

In 1973, Tobler introduced the idea of unquantized maps, without the generalization error resulting from the selection of class intervals, or quantization levels, in choropleth mapping. Tobler was dissatisfied with the use of a limited grey scale as the generalization solution to the tradeoff between accuracy and simplicity on choropleth maps. His main argument originated from theory on picture processing, which recommends up to 100 quantization levels for an acceptable quantized picture(Rosenfeld, 1969). Rosenfeld explains that a fine degree of quantization is particularly important if a region with slowly changing grey levels is being quantized. If the quantization is not fine enough, then there will be curves along which there is an abrupt grey level jump. These conspicuous curves "...may make the quantized

Figure II.3



approximation to the original unacceptable, since they define spurious 'objects', which may compete with or conceal the real objects shown in the picture"(Rosenfeld, 1969, p. 22) The 'false contours' of a coarse quantization in picture processing are easy to see, as are the simpler patterns of a coarsely generalized choropleth map. However, the accuracy of both types of quantized image is generally low, particularly if the original image contains a wide range of information. Tobler questioned the validity of the cartographer's use of a small number of classes in choropleth mapping, versus the high number of levels in picture processing, when the objective of successful transmission of accurate graphic information was common to both fields(Tobler, 1973). Examples of an generalized and unquantized choropleth map are seen in Figures II.1 and II.4, respectively. Since Tobler's original suggestion and arguments were set forth, there has been a growing controversy centered on the question of quantization on choropleth maps. Arguments and research have reflected the recent changes and developing theoretical framework in thematic cartography.

The Quantization Controversy

Initially, arguments against choropleth maps were practical, as well as theoretical. The coarse maps produced by line plotters and printers, for example, the map accompanying Tobler's 1973 article, were unattractive, with the hatching or overprint patterns causing considerable visual static. The development of the facsimile map, with its high design standards and ease of production, has negated arguments against unquantized maps based on design grounds. Now, the strongest arguments for and against unquantized maps are theoretical, dealing with the significance of classification in choropleth mapping(Muller, 1979).

The quantized map is conventional, and many arguments have been set forth in its favour. The quantized map allows the cartographer to highlight patterns, and ensure the desired interpretation of the information presented(Monmonier, 1977). Quantized mapping allows the identification of each category on the choropleth map(Dobson, 1973). Regionalization of the mapped variable permits a quick overview of the distribution(Monmonier, 1977). Also, the simplified pattern may be memorized easily by the map viewer. The proponents of the quantized map recognize the loss of accuracy inherent in generalization; however, the optimum choropleth map is defined in terms of a

Figure II.4

tradeoff between accuracy and simplicity, with accuracy assigned secondary importance.

The unquantized map is statistically accurate, because each element of the data is represented by a separate tone. The creators of the unquantized map are not concerned with providing a form of areal table for the map viewer. Rather, they are concerned with transcribing an order between cartographic signs, regardless of their individual meaning(Muller, 1980). The unquantized map is the furthest extreme in allowing the map viewer to establish his own meaning from accurate information presented on the map. The geographical accuracy of the unquantized map obviates misinterpretation resulting from an assumption that a mapping category is more homogeneous than it really is(Monmonier, 1977). A pattern contrived through the use of classification may be viewed as a "useful lie"(Muller, 1980), which may unjustifiably impose order where none exists.

Muller(1979) tested the ability of the map reader to generalize patterns from an unquantized map of 120 levels. The early work of Jenks and Knos(1961) tested the viewers' ability to identify individual shades on the choropleth map, while Muller attempted to simulate the basic task of using a choropleth map, by examining the viewer's success in identifying and organizing patterns in the mapped data. Muller discovered that viewer response generalizations of unquantized data approached optimum generalizations of the same data. Peterson(1979) tested the map viewer's ability to discriminate between shadings on quantized and unquantized maps. He discovered that subjects possess a remarkable capability for visual estimation of the values represented on an unquantized map, and that these values were conveyed to the map viewer more accurately by an unquantized map than by quantized maps with fewer than six shadings.

The current dispute over quantized versus unquantized maps has clouded the central issue in choropleth mapping; the establishment of a quantization level which represents the optimum tradeoff between accuracy and cartographic communication on choropleth maps. The findings of Muller and Peterson should not be viewed as attempted resolutions of this issue, but as an introduction to further research in this area. Research into discovery of the optimum quantization level will be conducted in the current context of technological change, and acceptance of the successful communication of meaningful cartographic information as the chief function of maps.

C. Aims of Research

It is the aim of this project to study the cartographic conventions in quantization, based on the hypothesis that an optimum generalization solution exists between quantization extremes which favour either accuracy or simplicity in choropleth mapping. This solution would maximize a combination of accuracy and simplicity, allowing the map user to construct a meaningful spatial picture of the original statistical information. The framework for this research is comprised of specific findings in research into quantization in choropleth mapping, recent technological developments in cartography, and the currently accepted paradigm of cartographic communication.

III. Methodology and Procedure

A. Introduction to Research Design

The methodology and procedure applied in this study are the first stages in an overall research design. An outline of this design is as follows:

1. initiation of inquiry
   a. problem observation and definition.
2. approach to solution of observed problem
   a. derivation of hypothesis based on observed problem, framework of past research, and cartographic communication.
   b. collection of data as information base for problem analysis.
   c. data processing and analysis to establish patterns and relationships in data structure.
   d. graphic representation and description of observed patterns and relationships in order to obtain a clear image of findings.
3. Conclusions
   a. make conclusions about relationships in data structure with reference to hypothesis and initial problem.
   b. use conclusions to place study in framework of current research and trends in thematic cartography, and to raise questions about future direction of research in choropleth mapping and the development of cartographic communication theory.

Methodology and procedure incorporate the phases of initiation of inquiry, hypothesis derivation, and data collection in the research design.

B. Methodology

Problem Observation and Initiation of Inquiry

Initiation of this inquiry was triggered by the observation of a problem in choropleth mapping, a specific area in the field of thematic cartography. The problem has been defined, essentially, as the controversy regarding the search for an optimum generalization solution that balances the inevitable tradeoff between accuracy and

cartographic communication, which occurs when numerical ratio information is transferred to the graphic form of the choropleth map. Implicit in the observation of this problem is the rejection of traditional beliefs, or the purely negative procedure of doubting(Northrop, 1947). The approach is particularly appropriate in this situation, since a key element in the observed problem in choropleth mapping is the presence of traditional cartographic conventions in the context of rapid technological and theoretical changes in thematic cartography. In the absence of a general theory of cartography, cartographic conventions in choropleth mapping may be denounced with ease. Unfortunately, the absence of a sound theoretical framework may also prevent a proposed solution to the quantization controversy from being viewed as anything more powerful or justifiable than the original cartographic conventions themselves.

Method of Investigation

Two fundamental methods of scientific investigation have been identified. The deductive approach is based on a carefully and skilfully established framework. It is often referred to as the theoretical approach, since the deductive process involves preliminary exploration of all the theoretical ramifications of a problem, and the construction of a model which determines the logical sequence of steps in the research. The inductive, or empirical, approach consists of research conducted without the discipline implicit in a strong theoretical base. The inductive form of investigation consists of establishing a universal proposition, based on the measurement and classification of ordered facts(Norcliffe, 1977). The essential advantages of the deductive approach over the inductive approach are the comparative and predictive powers of theory. Researchers using a deductive methodology have the potential to isolate initial assumptions and common traits when examining new problems within the same theoretical framework(Beaujeu-Garnier, 1976). An inductive methodology may be used to advantage when a unique or personal approach is deemed appropriate for studying a particular problem. It has been argued that inductive and deductive approaches differ in degree rather than kind(Norcliffe, 1977). This argument is supported by the fact that perfect induction, in which all instances under the established universal proposition are exhausted, is an example of a deductive argument(Northrop, 1947). The central guideline for this research is based on the premise that deductive and inductive modes of investigation are

not antithetical.

The absence of a general theory of cartography pre-empts the application of a purely deductive methodology in the study of cartographic problems. However, the absence of theory does not pre-empt an approach to cartographic research which includes elements of both deductive and inductive methodology, as advocated by Harvey(1969) in his discussion of the early development of a science. Research conducted with the observation of concrete facts, and the awakening of a disciplined scientific curiosity, may promote the development of a general theory that may be used as a starting point for deductive reasoning in subsequent research(Beaujeu-Garnier, 1976). A combination of methodologies appears to provide a more rigorous base for procedure in a developing science, such as cartography, than the inductive method alone. However, limitations to the predictive and comparative capabilities provided by the deductive element of the approach must be recognized and accepted.

Three main sources form the methodology for this research. At a theoretical level, the paradigm of cartographic communication is used as the chief guideline. This theoretical guideline provides discipline for the methodological approach, and potential for strengthening the theoretical structure in cartography with the study results. Aspects of information theory and cognitive psychology, as they apply to cartographic communication, were also used as methodological guides, on the premise that "...researchers who refuse to borrow at least parts of theories from other fields put themselves at an unfortunate disadvantage."(Bunge, 1966. p.5) Finally, the study procedures of previous research in choropleth mapping were evaluated according to the theoretical criteria, and examined for applicability to methodology and procedure development. The three elements were combined to form a methodology which is suited to the observed problem in choropleth mapping, as well as to the development of a general theory of cartography.

Hypothesis

A hypothesis was formulated as the initial procedural step in approaching the problem of the quantization controversy in choropleth mapping. The hypothesis was developed within the framework of the proposed methodological approach; a

combination of the current atmosphere in thematic cartography, the currently accepted paradigm of cartographic communication, and the conclusions of recent research in the area of choropleth mapping. The hypothesis from which the experimental procedure of this study was formulated may be stated as follows: A threshold quantization level, which represents the optimum tradeoff between statistical accuracy and cartographic communication of the mapped information, exists for choropleth maps.

C. Procedure

Experiment Rationale

Preliminary Considerations

The experiment in this project was designed to test the proposed hypothesis. Two chief factors were recognized and defined for the study: map accuracy, and cartographic communication.

Map accuracy is a controllable factor, dictated by the quantization level of a choropleth map. Cartographic communication is a more complex factor, which will be defined in detail for the purposes of this study.

The purpose of choropleth mapping must be defined according to the general definition of cartographic communication as the transmission of meaningful information via the map. The basic purpose of choropleth mapping has been identified as the graphic communication of areal patterns of ratio distributions. In the context of the currently accepted paradigm of cartographic communication, successful cartographic communication in choropleth mapping may be defined by the viewer's ability to perceive distribution patterns in the mapped information at a given quantization level. The meaning of the information on the choropleth map is defined, therefore, by the viewer's perception of that information. It is the application of the general definition of cartographic communication to choropleth mapping, combined with recognition of the basic factor of accuracy as the key influence on cartographic communication, which forms a base for the experimental and analytical design of this thesis.

Experiment Design

Approach

The experiment was designed to function as a means of establishing an information source which could be used to study cartographic communication in choropleth mapping. Using the application of cartographic communication to choropleth mapping as a base for experiment design, a test was constructed to collect data about the map viewer's extraction of meaningful information from a variety of choropleth maps possessing a range of quantization levels. This data could then be examined and analyzed to assess the effects of map accuracy on cartographic communication in choropleth mapping. The final stage would consist of evaluating the final results in search of an optimum quantization level for choropleth maps. The basic approach of the experiment necessitated three steps: the selection and construction of a standard test map, the selection of a sample of map viewers, and the design of a test by which the required information could be gathered.

Test Map

Selection

The standard test map from which the series of test maps was constructed is a choropleth map representing the number of apartments per number of occupied dwellings in the city of Toronto, mapped by enumeration unit. The data was obtained from the 1976 Census for Toronto, published by Statistics Canada in 1978. The selection of the original map was undertaken within the guidelines of the following criteria, most of which were defined with the objective of maintaining the quantization, or accuracy, level as the chief source of complexity in the maps. The criteria are:

1. The area must possess a high number of enumeration units, in order to provide potential for a broad range of quantization levels in the series of test maps.
2. The enumeration units should be relatively small, to allow reasonably good definition in the mapped pattern.
3. The enumeration units should be consistent in size, to allow density levels and

their distribution to dictate the mapped pattern.

4. A map of the original unquantized data distribution should possess several nodes of high and low density, indicating a well differentiated distribution pattern within the original data.

5. The area covered by the test map should be unfamiliar to the test subjects, thus precluding viewer response based on a preconceived idea of the mapped distribution.

6. As a standardization measure, the series of test maps possessing a range of quantization levels should be constructed from the same original data set, using the same quantization procedure.

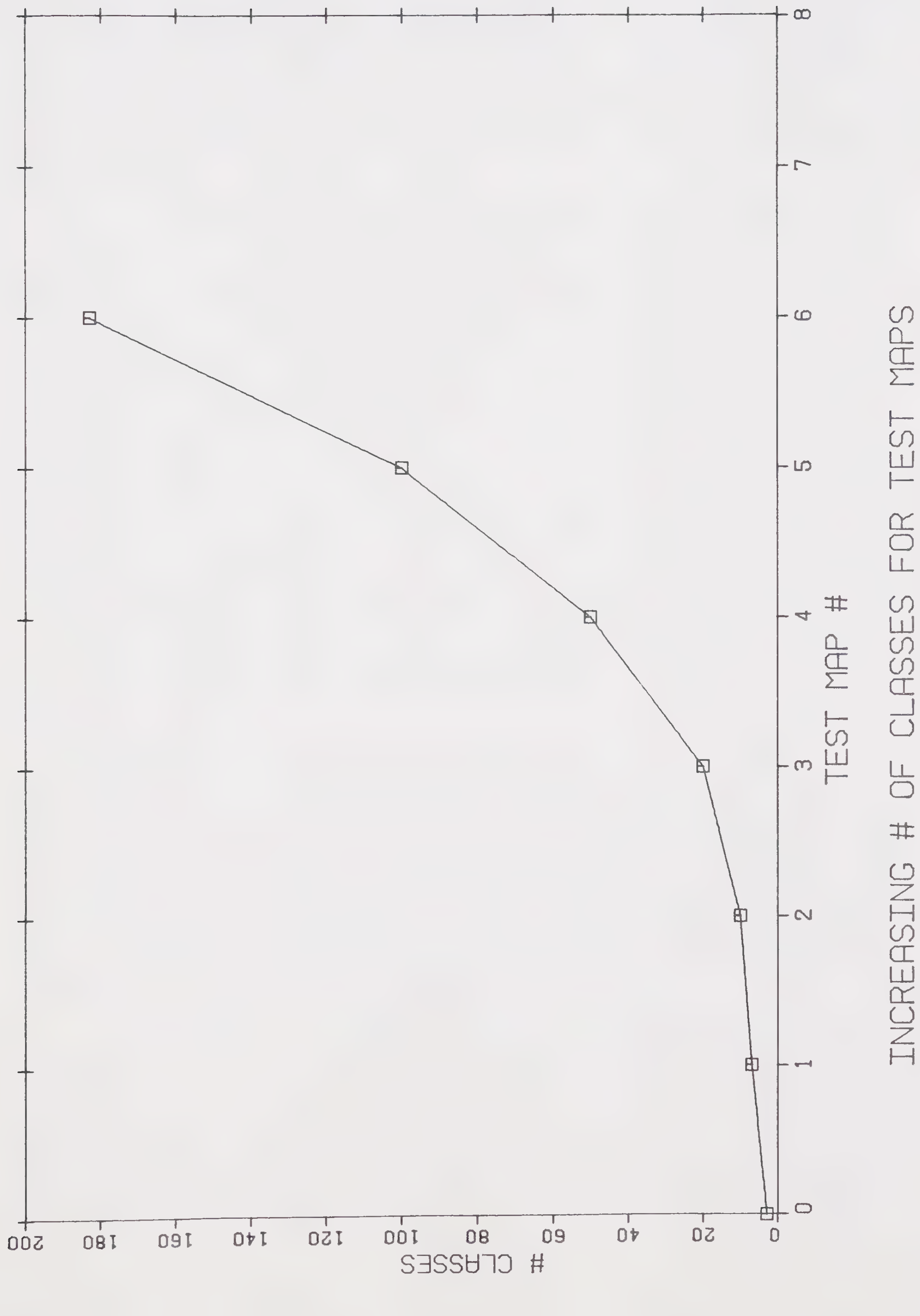
7. The possibilities of practical applications of choropleth mapping in urban planning, with the broad selection of readily available enumeration data, were also considered in selection of the original test map.

Construction

After the original test map selection, a series of test maps was constructed. The first aspect of test map construction consisted of choosing a series of quantization levels to be mapped for testing. The limits of the range of quantization levels were, essentially, extremes: an unquantized map displaying 183 density levels, and a highly generalized map displaying 7 density levels. The other maps were chosen to cover a range between quantization extremes, incorporating the anticipation of more subtle distinctions at the lower levels of the quantization spectrum.(see Figure III. 1) Due to volume restrictions in data, and time restrictions in data collection, only six quantization levels were chosen as a representative range: 183, 100, 50, 20, 10, and 7 levels. If deemed necessary, the test could be altered to accommodate a greater number of levels for added precision.

The second step in map construction was the selection of a standard quantization method to be used for the six test maps. Jenks' optimum classification program, which minimizes the absolute differences between original and mapped values for each class on the map at a specific quantization level, was used to generate the series of density values.

Figure III.1


INCREASING # OF CLASSES FOR TEST MAPS
TEST MAP #
CLASSES
0
1
2
3
4
5
6
7
8
0
20
40
60
80
100
120
140
160
180
200

The final step was the selection of a technical means of map construction. The maps were created using the facsimile mapping technique(Muller and Honsaker, 1978), producing density level representation of high quality, even for the 183 level unquantized map. The original values were scaled to a range from 0 to 255, based on the original range of 0 to 100, for input to the plotter. This new range was kept for the remainder of the study, since it represented the values to which people responded.

The original facsimile maps were reproduced photographically for testing purposes, in an attempt to maximize consistency within and between map sets. The decline in visual quality of the mapped pattern observed in photographically reproduced maps has been discussed by Monmonier(1979). He suggests that the data in a photographically reproduced map may be "unintentionally classed" by reproduction noise resulting from growth of the inked area. This visual distortion is definitely apparent in the test maps employed in this study. However, with the major considerations of the reproduction method defined as pattern consistency between maps, relative ease of production, and reasonable cost, photographic reproduction was still seen as the best method available for creating the test maps. The issue of visual distortion in choropleth maps which have been reproduced before publication will be considered in the conclusions of this project.

Sample Selection

Attempts were made to control the sample of map viewers used in the experiment. The sample was selected from what may be termed a potential map audience for this type of map. This approach was undertaken in accordance with the belief that the cartographer should consider the audience for which the map is being constructed when he makes design decisions. The geography students tested had similar backgrounds in geography, and common familiarity in the use of maps.

A single sample would not suffice for the set of six maps, since repetition of the experiment six times for each person might alter response patterns between the first and last map tested per person. Therefore, the overall sample was subdivided into six groups, a subsample for each of the six test maps. Thirty people were chosen for each subsample, meeting the minimum requirement for statistical analysis within subsamples.

Generalization Task

Thirty test maps were created for each quantization level selected. A clear acetate coating, suitable for marking with a permanent felt marker, was attached to each map. A brief questionnaire was handed out with each of the test maps. Copies of questionnaire and test maps are included in Appendix A. In the questionnaire, respondents were instructed to generalize the density pattern displayed on the map into three categories of high, medium, and low density, by delimiting the map into these groups using the felt marker. Procedure was standard for all six map sets.

A pilot study was conducted to evaluate the questionnaire and procedure. The original questionnaire was then altered to include a statement that the lightest areas on the map were those of lowest density, and the darkest areas were those of highest density. In the final testing, 180 response maps, representing the subjects' generalizations of the mapped information, were collected.

Data Organization

The collected generalizations were transferred to numerical form for analysis purposes. Each subsample was organized separately in a 30x183 matrix of response values. A sample showing the structure of this matrix is found in Appendix D. Each row represented the thirty individual responses of high, medium, or low for a particular enumeration unit on the map. The responses were assigned the following values:

1. High−3
2. Medium−2
3. Low−1

Each column represented an individual respondent's generalization for the entire test map.

The median response value was calculated for each unit on each of the test maps, based on the thirty responses for that unit. The representative median map upheld the ordinal level at which the responses were collected. The representative median maps for each subsample were used to evaluate similarities and differences within and between subsamples.

D. Operating Definitions in Data Analysis

The terminology used in data analysis is as follows:

1. Units– the enumeration units on the original map of Toronto.
2. Ranked original units– the enumeration units from the original map of Toronto, ranked in ascending or descending order, according to density value.
3. Sample– entire sample of 180 responses.
4. Subsample– any one of the six sets of thirty responses to the test maps.
5. Tests maps– the six original Jenks' classified maps to which the the subjects responded.
6. Median response maps– the six representative median value maps compiled for each subsample.
7. Original unit values– those density values displayed on the unquantized map.

IV. Analysis and Results

A. Introduction

This chapter is a summary of the third phase in approaching a solution to the quantization controversy in choropleth mapping; data processing and analysis in an attempt to discover patterns and relationships internal to the data structure, complemented by graphic representation of results to obtain a clearer picture of these patterns and relationships. Investigation of the effects of map accuracy on cartographic communication in choropleth mapping was undertaken through comparison of original data, viewer response maps, and median response maps at several levels. Essentially, it was expected that a comparison of viewer response patterns within and between subsamples would provide a strong indication of the source and significance of response variation. The series of results would be used to make conclusions about the effects of quantization and complexity on the map viewer's ability to generalize the information, or create regional patterns, on a choropleth map. The results might also provide support for the hypothesis that an optimum quantization level exists for choropleth maps.

B. General Approach

The approach to data processing and analysis consists of four distinct, yet closely interrelated, levels. At the first level, major characteristics of the original data set and test maps are identified and described, in order that they may be used as a standard for the comparison of response maps within and between subsamples. At the second level, the six sets of thirty response maps for each of the subsamples are examined individually, so that relationships and patterns between responses within subsamples can be defined. These results then become a second standard for the evaluation of differences between the six subsamples. The third level involves comparison of the observed relationships and patterns within subsamples, in order to establish a further set of patterns and relationships between subsamples, which will provide an overall picture of response variations between between different test map quantization levels over the entire sample. Finally, the observed patterns at all three levels are compared, and combined to form a complete set of results. This set of results is the source from which conclusions about

the quantization controversy and proposed hypothesis were derived.

C. Limitations in Response Analysis

The data were collected in the form of three class response maps. Therefore, patterns in viewer response were measured according to the characteristics of response maps and median response maps, compared with one another, and with the characteristics of the original data and test maps. A set of criteria for response measurement and comparison was compiled, and techniques for deriving the measurements from response maps were formulated.

Measurement and comparison of responses were restricted by the ordinal level at which response maps were constructed, and, more significantly, by the areal factor inherent in a two dimensional response map. Each response consisted of an array of 183 values, with each value representing an ordinal response for a particular unit on the original test map. Attempts were made to apply standard statistical tests designed for ordinal data, such as the Kruskal–Wallis test and median test, with little success. While values were obtained using these tests, it was apparent that the tests failed to incorporate the distributional qualities of the responses, and could not be presented as valid representations of response variation. Thus, it was necessary to design selected measures of viewer response which would provide information about patterns and relationships in two dimensional responses. This task was accomplished, and although the results obtained through the application of selected techniques lack the predictive power of results obtained through the use of accepted statistical analysis, they fulfill two important requirements in this study:

1. revelation of distinct patterns and relationships in the data structure.
2. provision of a statistically legitimate representation of an unusual and specialized data set.

D. Data Analysis

Selected Characteristics for Response Measure

The general characteristics of viewer response were selected to obtain detailed information from the data that would define patterns and relationships specific to the

observed problem and proposed hypothesis in this study. As chosen indicators of viewer response, these characteristics became guidelines for the choice and construction of data processing techniques.

Three basic characteristics were chosen: response accuracy, response consistency, and response unity. Response accuracy was chosen to define the viewer's ability to recreate a three class generalization of the data, as compared with a three class generalization obtained using Jenks' optimum classification, and with the three class response generalizations created at all six quantization levels. Response consistency was chosen to provide a measure of the general patterns of response among individual responses for each subsample. The degree of consistency between response patterns and the original data patterns would indicate those test maps with which people had the most difficulty. Response unity was identified as the level of agreement in response among individual responses for each subsample. Unity of response was recognized as a means by which source areas of confusion on the test maps, and overall levels of confusion within and between subsamples of responses, could be located. The measures of response accuracy, consistency, and unity were used jointly to provide a comprehensive measure of viewer response patterns, both within and between subsamples.

The results of response analysis were used in a comparison with original data, test maps, and three class optimum generalization. The characteristic of map accuracy, varying with quantization level, had already been selected as the most significant factor affecting cartographic communication in choropleth mapping. Map complexity, which is closely related to map accuracy, and affects the visual appearance of a mapped distribution, was selected as a secondary characteristic to be studied in the evaluation of cartographic communication. Map accuracy is a controllable factor, dictated by the quantization level of a choropleth map. Complexity, however, is an intricate feature, which is more difficult to control: it is related strongly to the quantization level and method used in making the map, as well as to the distribution features and enumeration areas associated with the original data. The complexity of a mapped pattern at a given quantization level is dictated by the distribution characteristics of the original data, and cannot be altered without alteration of the quantization method or the quantization level.

Complexity is defined by the contrast in density between adjacent units on the map, coupled with the level of visual disruption created by irregularities in size and shape of the mapped units. For example, a checkerboard contains a high level of contrast, but the regularity in size and shape of the units create a visually simple pattern. A high degree of complexity exists when there is a strong contrast between adjacent units on the map, combined with a high level of visual disruption in the size and shape of mapped units. Depending upon the nature of the original data, complexity may be, when combined with the added volume of information inherent in increased accuracy, a detrimental force in the transmission of meaningful information via the choropleth map: Two maps of identical areas and different data sets may be quantized at the same level of accuracy, yet the patterns of complexity for each map may be vastly different.

Once the desired characteristics of original data and response maps had been identified, the problem of designing techniques to measure these characteristics through processing of mapped data was approached. Several basic data processing techniques were designed and implemented using varying forms of the data structure, to permit comparative analysis of results at the four defined levels.

Data Processing Techniques

Introduction

The data processing techniques used in this study consist of computer programs, both original and previously used, which were designed, or modified, to manipulate arrays of numbers whose values were matched with the 183 enumeration units on the original map of apartment density in Toronto. The methods used to measure response accuracy, consistency, and unity, as well as those used to define characteristics of the original data and test maps, will be outlined in detail. A listing and documentation of the actual programs used in data analysis are included in Appendix B.

Response Accuracy

Overview Accuracy Index

Response accuracy was defined by characteristics of the three class response maps. The accuracy indicator chosen in this study is the overview accuracy index of response maps, calculated using the method set out by

Jenks and Caspall(1971).

Description of the overview accuracy index technique begins with the basic three dimensional data model used to visualize the characteristics of a ratio distribution. Each original distribution can be represented by a three dimensional data model. A generalized representation of the same distribution would be seen in a three dimensional generalized model. The volumetric space between the three dimensional generalized model and three dimensional original data model is the overview error factor of the generalization. The most accurate overview of a distribution will be obtained from the generalized model which most closely approximates the data model. It was this process which was used to create the six test maps used in this study. These generalizations, therefore, represent the highest overview accuracy possible at those generalization levels.

The volume of error is composed of a series of "error prisms"(Jenks and Caspall, 1971). There is one error prism for each unit on the map, with the error value representing the difference between the generalized value for the class in which that unit was placed, and the original data value for that unit. Some units in a class will possess a higher value than their original value, while others will possess a lower value. The magnitude of error for a generalization equals the absolute sum of the volume of all the "error prisms", which make up an overall blanket of error for the entire map.

Jenks and Caspall calculated an overview accuracy statistic, based on the measurement of overview error. In this case, the theoretical limits required in the development of such a statistic are defined by the most accurate and least accurate generalizations possible, that is, an unquantized map and a one class, ultimately quantized, map. Once these limits are established for a particular data set, the overview accuracy for any generalization of the data can be compared with that of any other. The overview accuracy index used in this study is equal to the error index subtracted from one. The formula for the error index is: total error volume of generalization/total error of most inaccurate map.

The overview accuracy index for a generalization, therefore, will be in a range from zero to one, with one representing a map which is one hundred percent accurate. In this project, the overview accuracy index was calculated for the six test maps, three class optimum generalization, median response maps, and individual response maps, permiting comparison at all levels of the data. The computer program used to calculate the overview accuracy index required the following information: number of units, number of classes in the generalization, densities of original data, size of enumeration units, and class limit densities for each generalization. A copy of this program is listed in Appendix B.

Complexity/Contrast Index

The next data processing technique was created to produce a measure of the complexity level associated with each quantization level, for each of the test maps. It was proposed that response accuracy could be better understood if test map characteristics that were associated with map accuracy were understood also. Patterns of changing complexity could be compared to patterns of changing accuracy for all test maps. The comparison would provide information about the relationship between accuracy and complexity for the maps, thereby allowing more comprehensive evaluation of sources and patterns of response variation. Contrasts in density levels among triads of adjacent units over the map were used as the general standard of map complexity. Based on this standard, a method for establishing a complexity/contrast index for each test map, and for additional quantization levels for which data were available, was fashioned. The indices were then graphed to exhibit a pattern of complexity varying with quantization level.

The first step in calculating the complexity/contrast index is the identification of all triads of adjacent units on the original map. This number will be the same for all test maps, while the density values for those units would vary between test maps. The definition of an adjacency triad is a situation where three units on the map are adjacent, with each unit being adjacent to the other two. A program was written to search a logical matrix

of dimension 183x183, constructed to cross-match adjacent units with a value of one, and those units which were not adjacent with a value of zero. The program was designed to produce a list of the adjacency triads. The list for this map contained 144 triads.

The second phase of calculation involves computation of the contrast levels within adjacency triads. The absolute differences in density value between three sets of two units were computed for each adjacency triad. These contrast levels were calculated and summed for each test map.

In order to define a complexity/contrast statistic, theoretical limits were set. The lowest possible contrast level is zero, occurring when all three density values are the same. The highest possible contrast level is 510, since the highest mapped value is 255, and the lowest is 0. Thus, minimum complexity would result if the contrast level for all triads was zero(144x0). Maximum complexity would result if all triads possessed the maximum contrast level of 510(144x510). If maximum contrast is used as the theoretical measurement standard, then a summation of the contrast levels for all 144 triads, divided by the maximum contrast, will yield a complexity/contrast index, the maximum of which would be one, and the minimum zero. The formula is as follows:

$$\frac{\sum(C_{ij} + C_{ik} + C_{jk})}{\text{max. contrast}}$$

where C=absolute differences between unit densities, and i,j,k=units in adjacency triad. Since the index is based upon a theoretical standard which could be applied to all of the test maps, it can be used to define and compare the complexity/contrast characteristics of all these maps. The programs used to calculate the index are listed in Appendix B.

Response Consistency

The measure of response consistency is based on a comparison of the unit rankings in the original data, and deviations from original data ranking seen in the median response maps. The original data distribution and test maps all possess the

same ranking of units, from lowest density to highest density, regardless of class values or class limits. However, the median response maps contain ranking errors, due to consistent misplacement of a unit into a class above or below its ranking order. Ranking errors were looked at in several forms:

1. disruption of distribution patterns.
2. volume of ranking error by quantization level.
3. volume of ranking error by ordinal class level.

The class limits for median response maps were set as the first occurrence of that value(either 1, 2, or 3) in the response array. Misranked units are those units contained within the class limits of another class value. No class in the original distributions will contain misranked units, while each class in the median response maps has the potential for containing misranked units. For example, the range of the lowest class may contain both medium and even high values. The level and pattern of this inconsistency would vary for each test map. The misranking of units also results in an overlap between classes, with the creation of gaps between the end of one data class and the beginning of another. Essentially, the original units were ranked according to response value, so that patterns of disruption in distribution for each response could be viewed. This pattern is displayed in graph form for each median response map. Absolute numbers of misranked units for each quantization level of test map were also graphed. Finally, misranked units were graphed for each class at each quantization level, so that the class level in which most errors were made, and patterns of this error over the different test maps, could be made.

Response Unity

Response unity was the third general characteristic chosen as a possible measure of viewer response. A method of defining unity according to the three class response maps was obtained by calculating the total number of deviations from the median response value for each unit on the map. For example, if the median response was 2, and there were twelve people who responded with either 1 or 3 for that particular unit, then the number of deviations for that unit would be assigned a value of twelve. The total number of deviations for a subsample was

calculated by adding the number of deviations for each of the 183 units to which viewers responded. The mean deviation value was derived by division of the total number of deviations by 183, to obtain a representative measure for each subsample. The mean deviation value was used as a measure of response unity. The program used to calculate response deviations is listed in Appendix B. A sample listing of deviations by map unit is found in Appendix D. Distribution of disagreement levels was calculated for each subsample, as a measure of unity within subsamples. Finally, the deviation levels were graphed for each unit on the map, from highest to lowest ranked original units. This was done in an attempt to locate the areal patterns of disagreement, and possibly define sources of response error through areal similarity between subsamples.

E. Results of Data Analysis

Introduction

The results of this study were obtained using the levels of analysis and techniques of data processing previously outlined. Patterns and relationships in viewer response are, basically, distributions over the range of test maps or by map unit. Patterns in the data are best shown by graphs because they permit observation of recurring or fluctuating patterns along the constant X axis covering the range of quantization for the six test maps. The graphs are also useful for the display of patterns along an axis of the original ranked units, to bring out variations associated with unit density level. The outline of results will be divided into three main sections, representing the process used in data analysis; characteristics of original distribution and test maps, results of analysis within subsamples, and results of analysis between subsamples, including an overall comparison with other results.

Characteristics of Original Data Distribution and Test Maps

Data Distribution

The data distributions for all six test maps and the three class optimum generalization were graphed, from lowest data value to highest data value by unit, so that alterations to the original distribution resulting from quantization of the data could be examined. Graphs of these distributions are displayed in Figures IV.1–IV.7.

Lists of original data are found in Appendix D. From these graphs, several interesting observations can be made. Moving from the original distribution towards the 3 class generalization, there are no readily apparent steps in the distribution until the 20 class level. This indicates a general lack of disturbance to the data distribution by the optimum generalization program until this generalization level is reached. This lack of disturbance is seen in these graphs, and also in the visual appearance of the data on the test maps. The general trend of the distribution is not significantly altered until the 10 class test map, on which the trend of concentration of data in the lower classes begins, and very large steps in the data become apparent. The trend intensifies at the 7 and 3 class quantization levels, and should be considered as a potential influence on viewer response to the test maps.

Complexity/Contrast Level of Test Maps

The complexity level of the test maps was considered a major factor in the study of transmission of the added information made available to the viewer through more accurate maps. Complexity depends strongly on the nature of the distribution, not solely upon the quantization level at which the data are generalized. The complexity/contrast index was calculated for the six test maps and the three class optimum generalization. The results of this analysis are displayed in Figure IV.8. A list of values is found in Table 1, Appendix C. The basic pattern of complexity versus quantization level consists of a steep rise beginning at the 3 class generalization level, a gradual rise to the 50 class level, and a complete leveling off at this point.

As might be expected with the method used to calculate complexity, the 3 class generalization possessed the lowest level of complexity, due to the presence of large areas with a zero contrast level in density. There is, however, a sharp rise in complexity with the 7 class generalization which is more difficult to explain, particularly because this is the highest complexity index for all six test maps, and because there is a drop in complexity level immediately after this value. It appears as though the 7 class generalization represents a threshold level of quantization for complexity. The data has been divided into distinct groups; however, the groups exist on the maps in small sections, as opposed to the large cohesive masses on

Figure IV.1

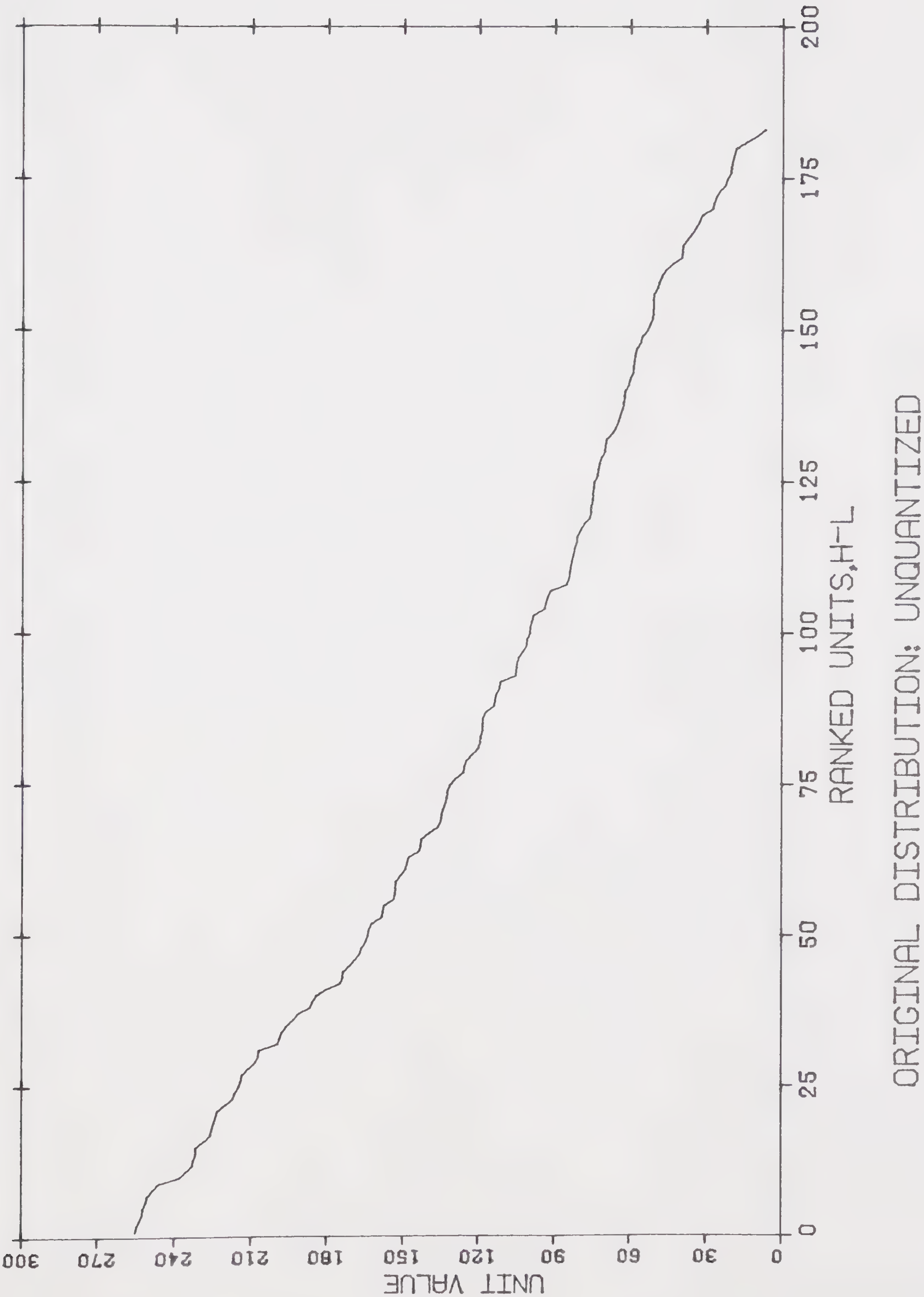

ORIGINAL DISTRIBUTION: UNQUANTIZED
RANKED UNITS,H-L
UNIT VALUE
0
25
50
75
100
125
150
175
200
0
30
60
90
120
150
180
210
240
270
300

Figure IV.2

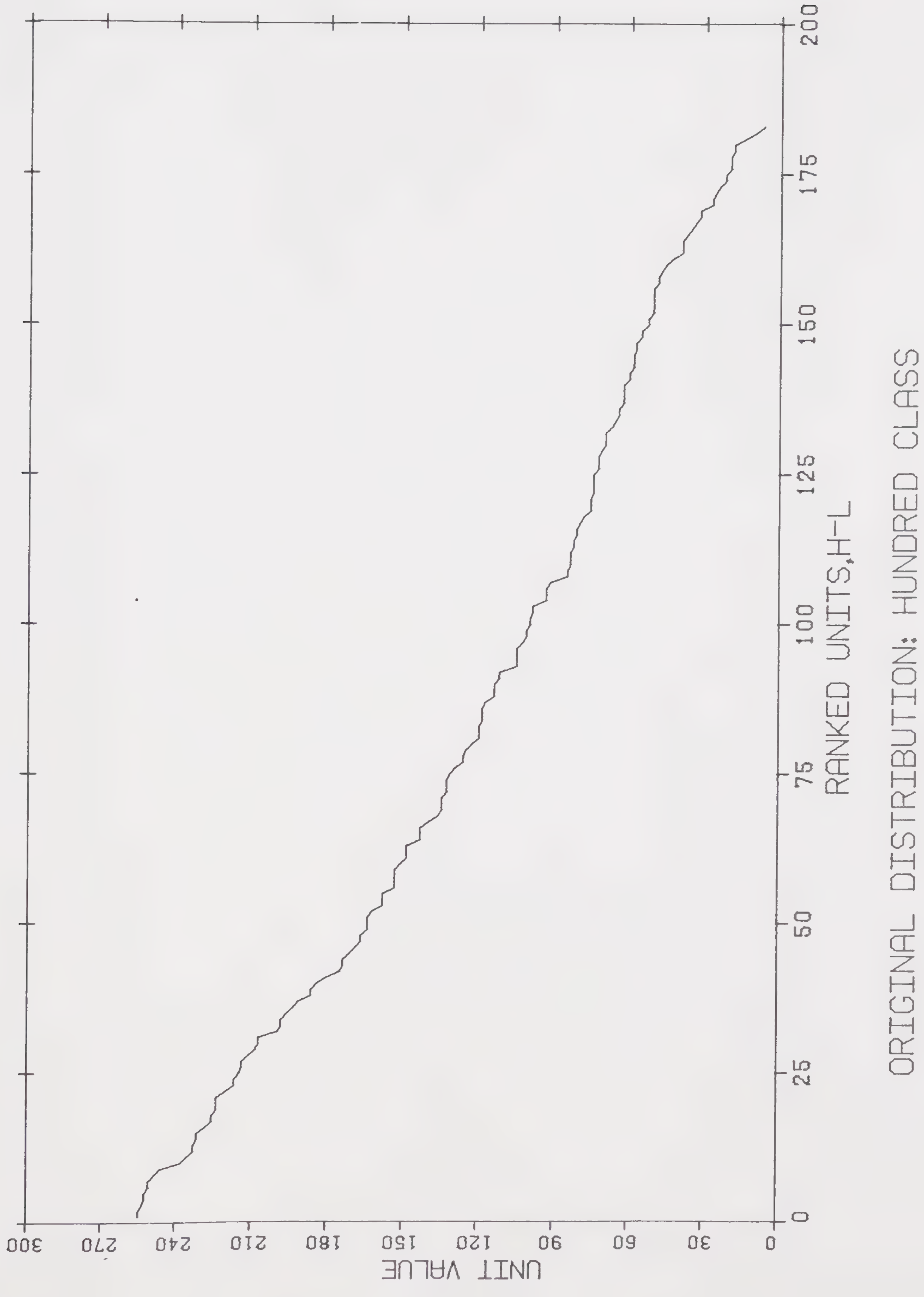

ORIGINAL DISTRIBUTION: HUNDRED CLASS
RANKED UNITS,H-L
UNIT VALUE
0
25
50
75
100
125
150
175
200
0
30
60
90
120
150
180
210
240
270
300

Figure IV.3

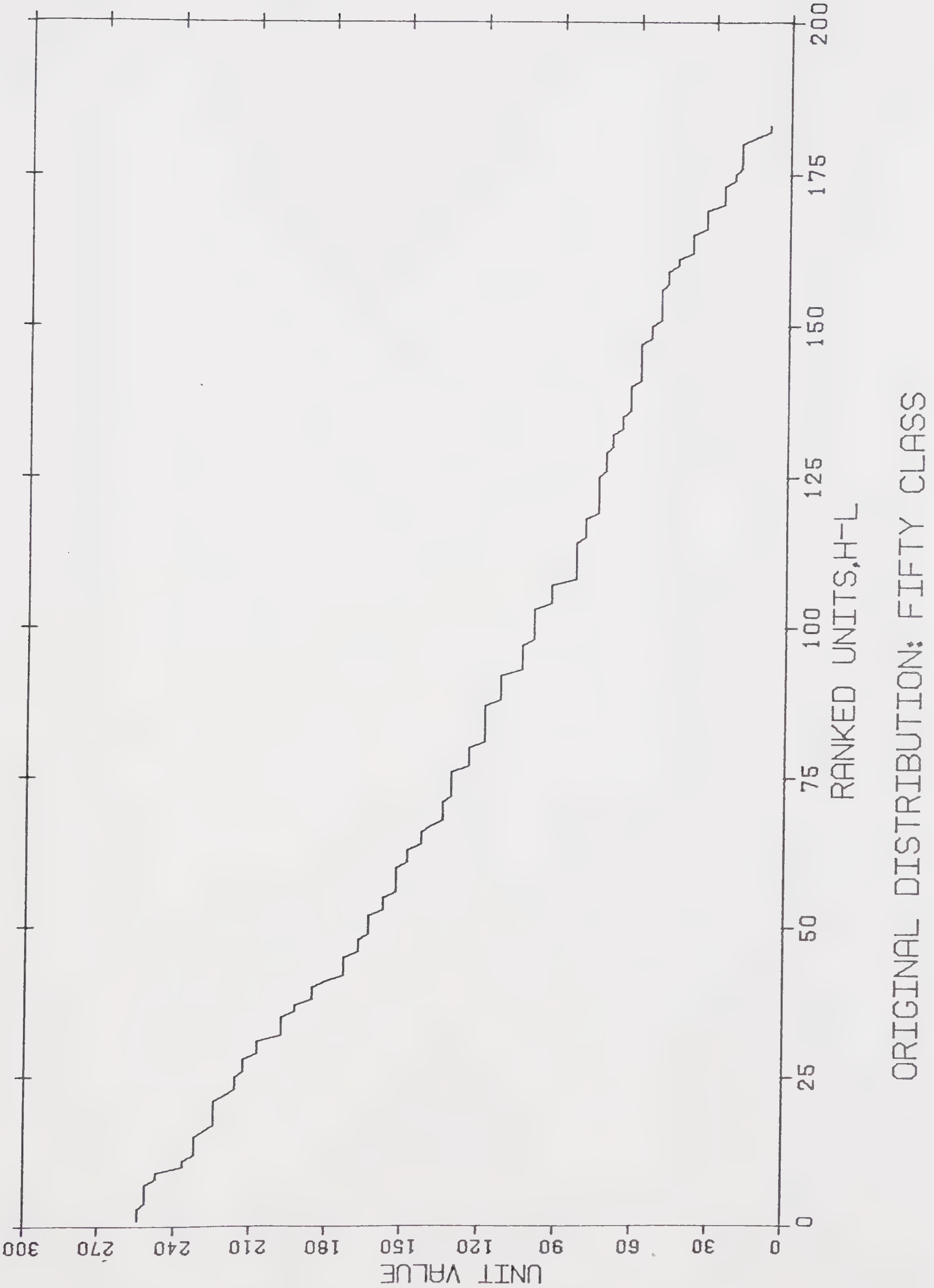

UNIT VALUE
0
30
60
90
120
150
180
210
240
270
300
RANKED UNITS,H-L
0
25
50
75
100
125
150
175
200
ORIGINAL DISTRIBUTION: FIFTY CLASS

Figure IV.4

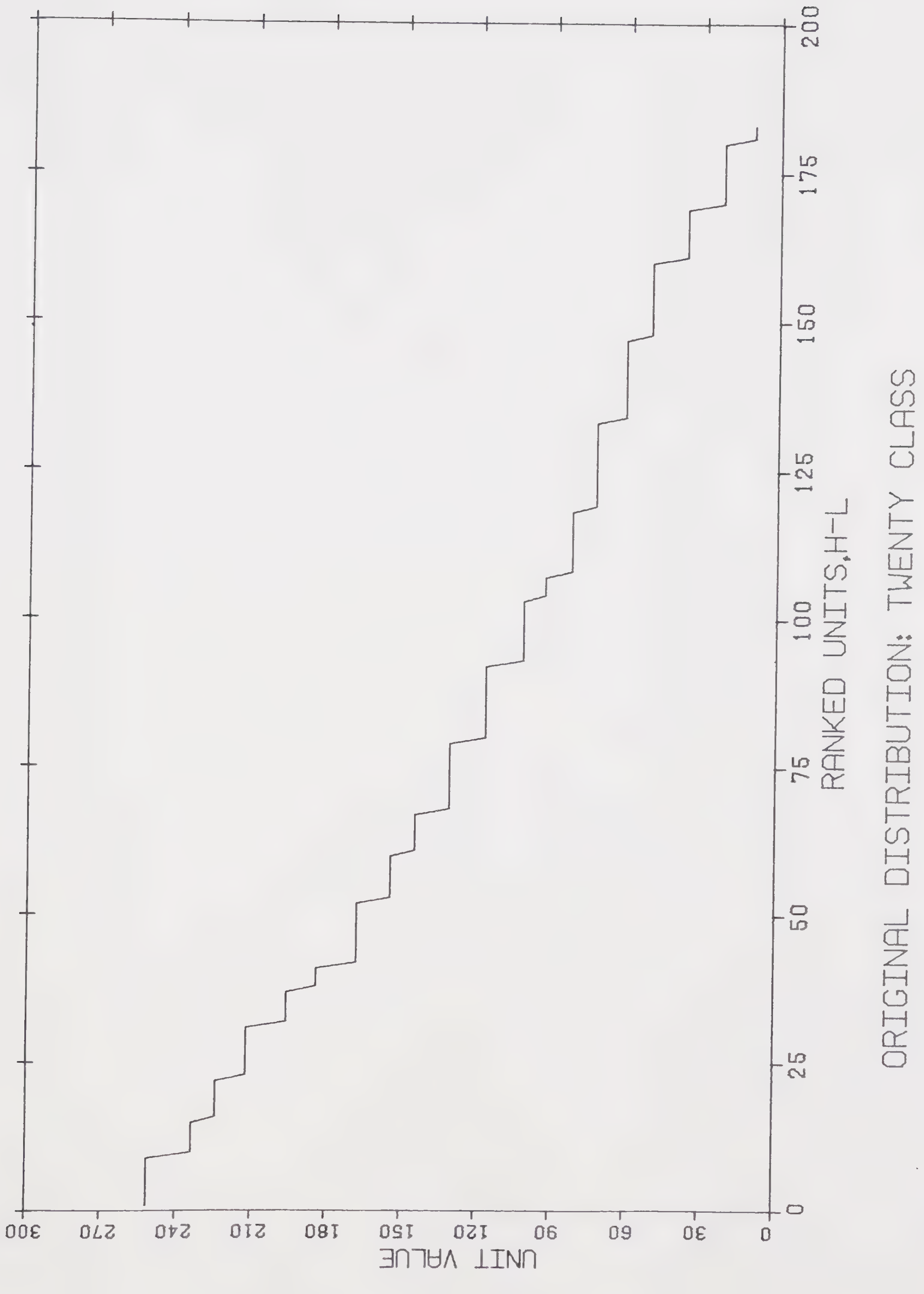

ORIGINAL DISTRIBUTION: TWENTY CLASS
RANKED UNITS,H-L
UNIT VALUE
0
25
50
75
100
125
150
175
200
0
30
60
90
120
150
180
210
240
270
300

Figure IV.5

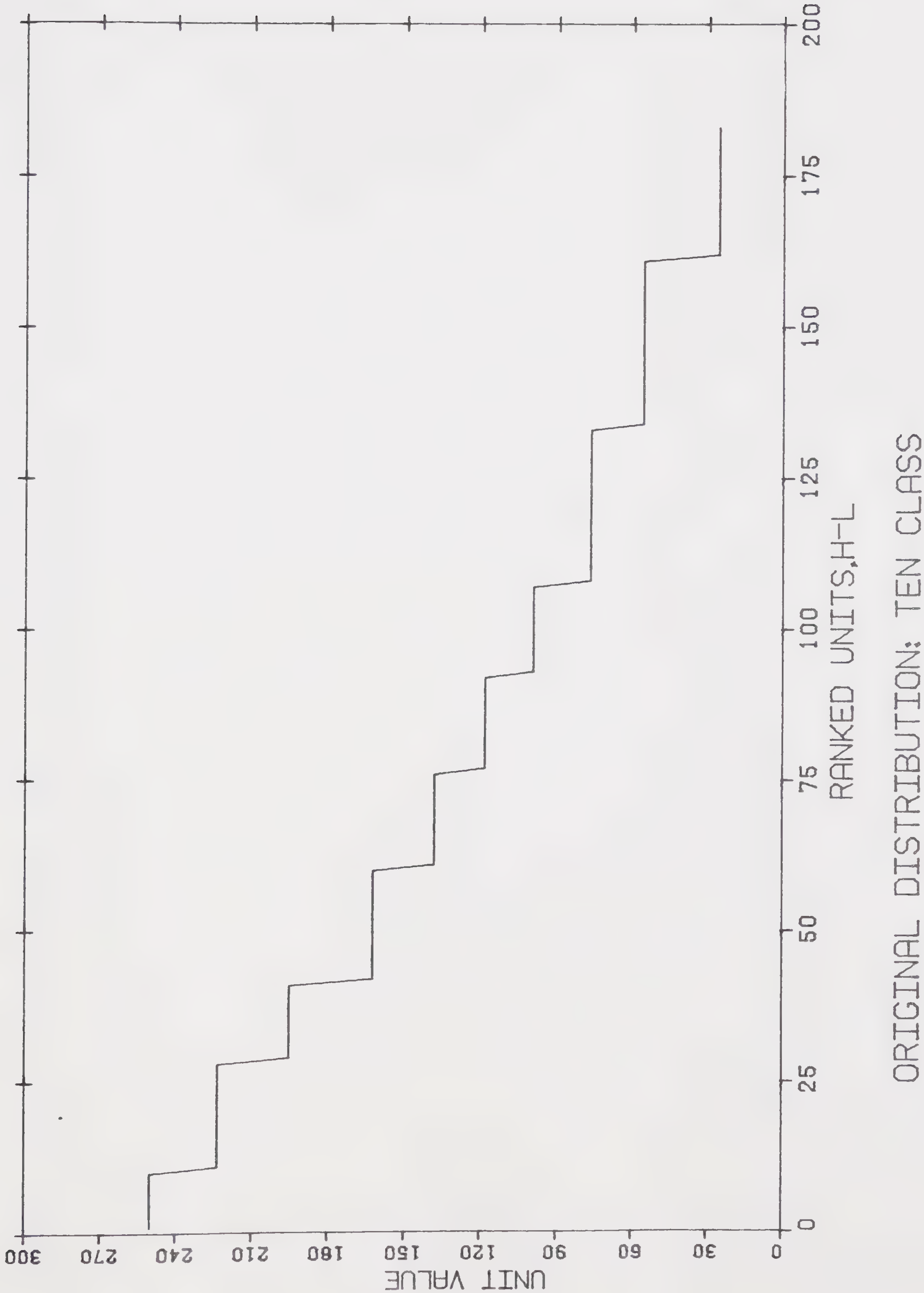
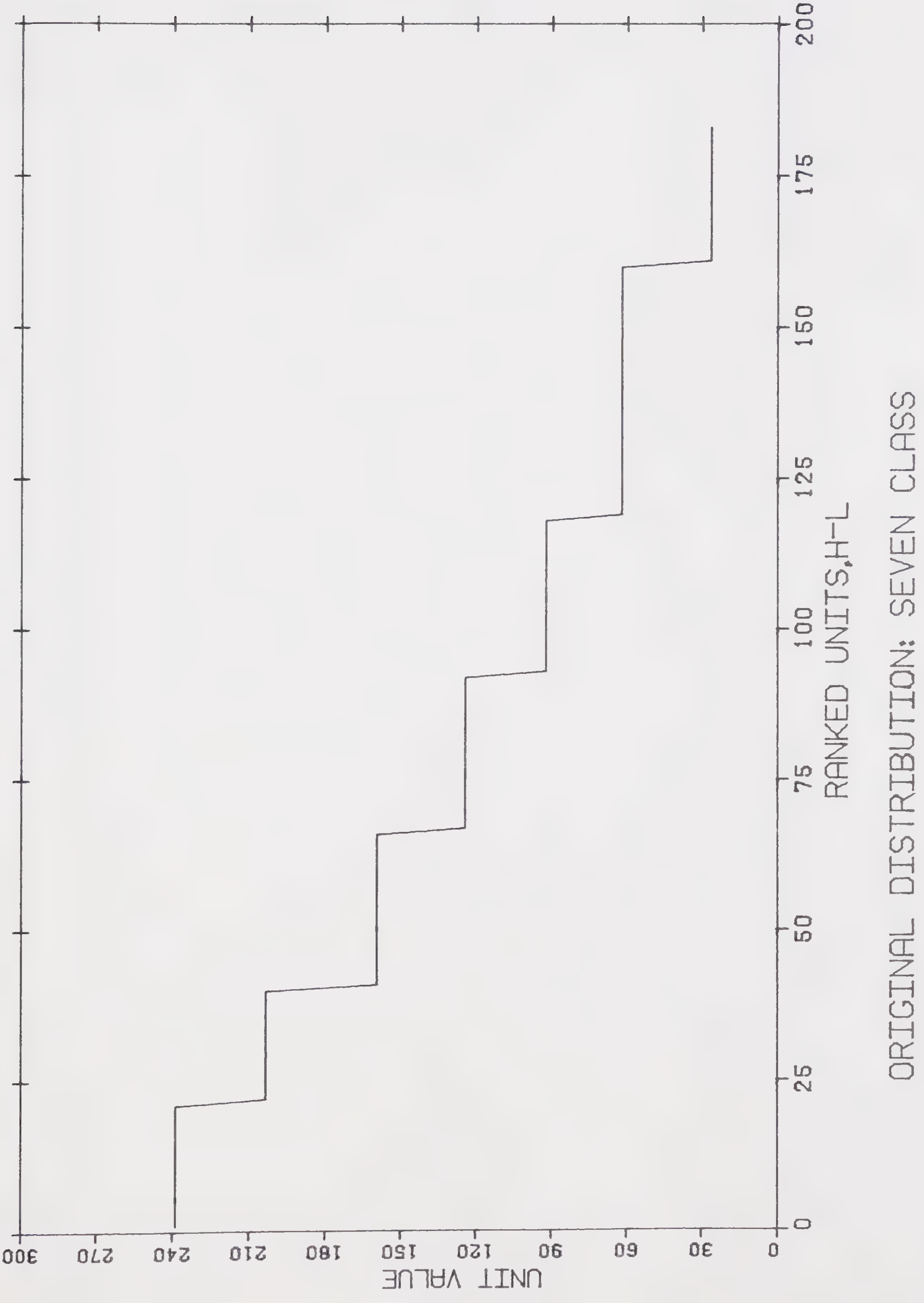

ORIGINAL DISTRIBUTION: TEN CLASS
RANKED UNITS,H-L
UNIT VALUE
0
25
50
75
100
125
150
175
200
0
30
60
90
120
150
180
210
240
270
300

Figure IV.6



UNIT VALUE
0
30
60
90
120
150
180
210
240
270
300
RANKED UNITS,H-L
0
25
50
75
100
125
150
175
200
ORIGINAL DISTRIBUTION: SEVEN CLASS

Figure IV.7

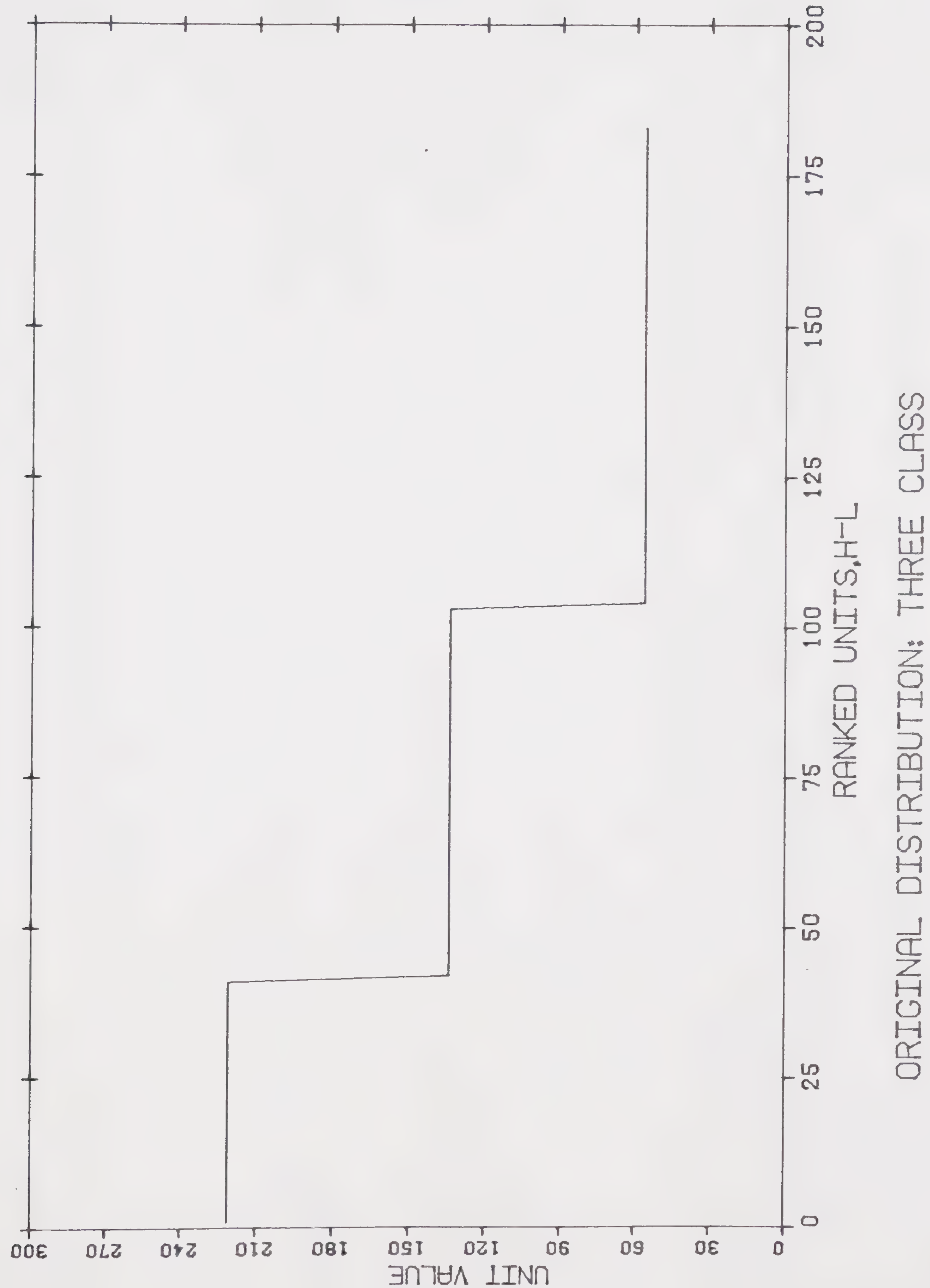
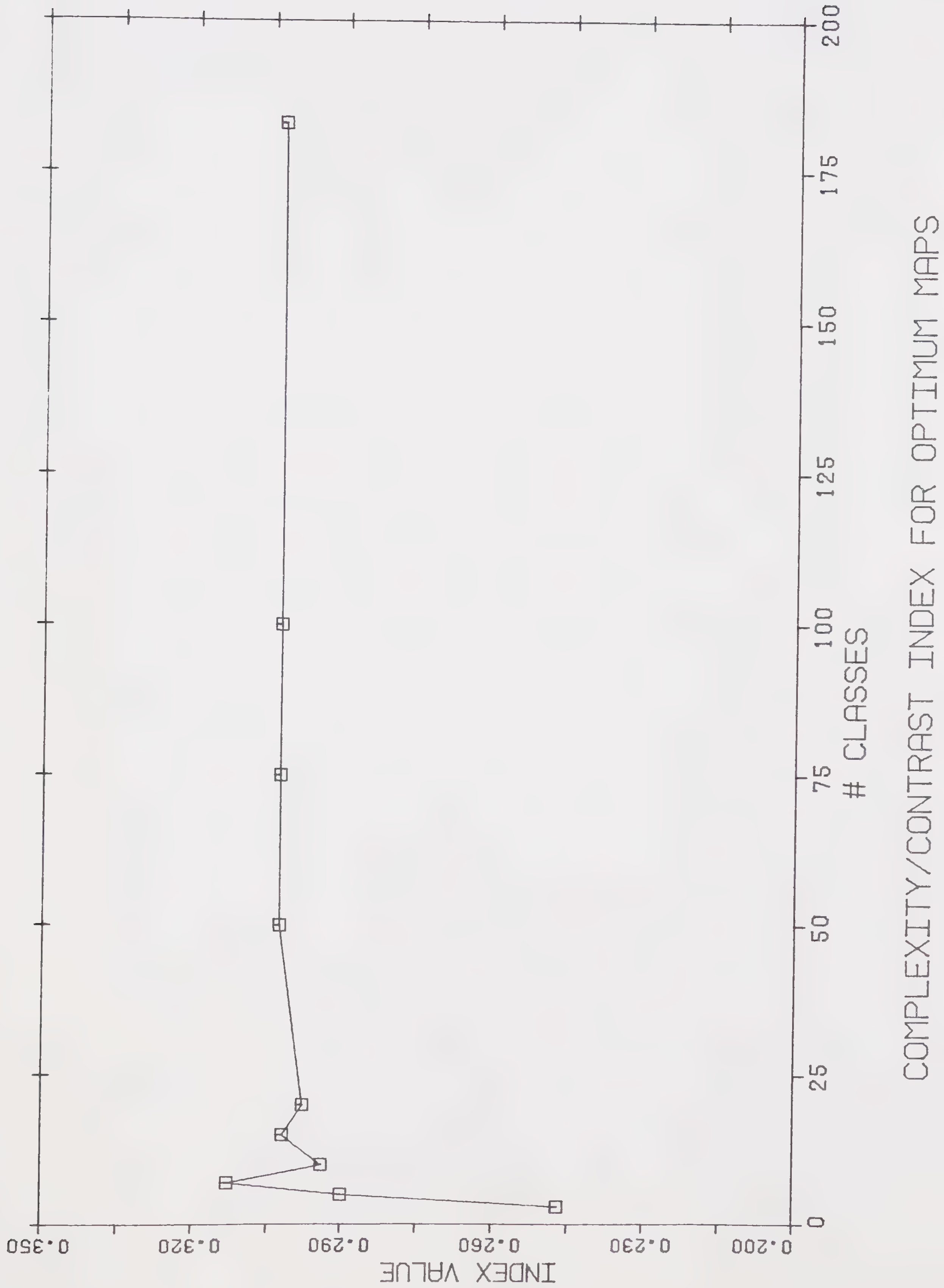

ORIGINAL DISTRIBUTION: THREE CLASS
RANKED UNITS,H-L
UNIT VALUE
0
25
50
75
100
125
150
175
200
0
30
60
90
120
150
180
210
240
270
300

Figure IV.8


COMPLEXITY/CONTRAST INDEX FOR OPTIMUM MAPS
CLASSES
INDEX VALUE
0
25
50
75
100
125
150
175
200
0.200
0.230
0.260
0.290
0.320
0.350

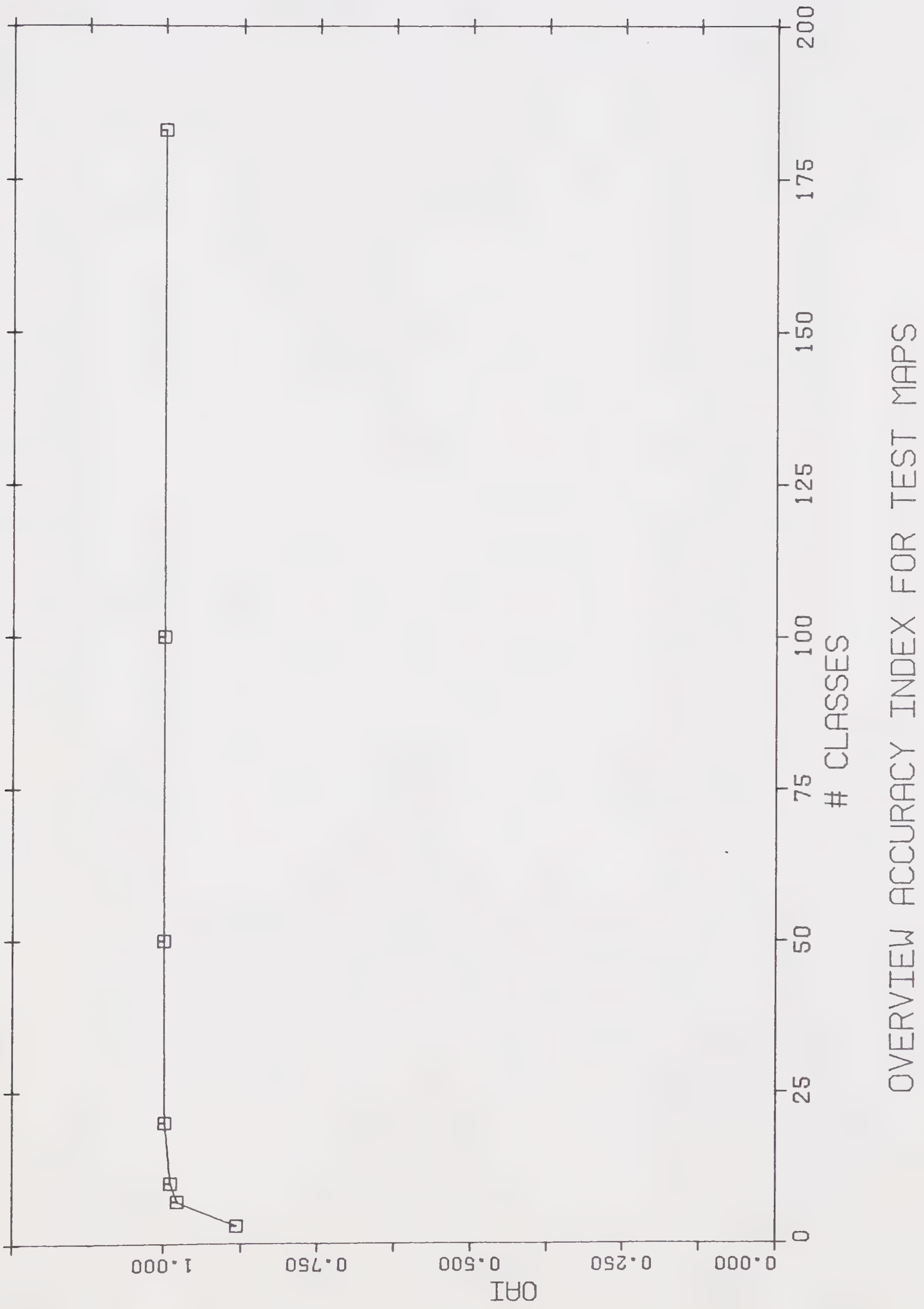
the 3 class map. The fine gradations from high to low density on the 10 to 183 class maps are not apparent on the 7 class test map. Rather, the contrasts in density are high over the map, due to the jumps between density groups within the generalization, and lack of spatial continuity in these groups over the map. The discovery of this pattern reinforces the fact that generalization disrupts natural patterns in distributions. To take the argument slightly further, the discovery also provides an indication that disruption may occur at a particular threshold generalization level to the extent that a more complex map may result from a generalization that was originally designed to simplify the distribution.

It is the general pattern of complexity which is most important to this study. The range of complexity indices over the seven generalization levels is quite small, from .247 to .303. This indicates a consistency in the distribution pattern of the original data which is maintained, in general, through generalization from 100 down to 3 classes. The narrow range in complexity, and strong consistency in distribution pattern may be the result of a high level of spatial autocorrelation in the areal pattern of the data. The general pattern of complexity variation shows a more significant change at the lower levels of highly generalized maps, with the addition of accuracy at the higher levels having absolutely no obvious effect on map complexity. The observed narrow range of low complexity indicates that complexity will not be a significant explanatory element in evaluation of response variation for this study. However, general and specific pattern trends of complexity have led to observations about generalization, and about variations in complexity with changes in map accuracy. These observations may be applied in other perception studies, and in the examination of map accuracy and complexity in the practical situation of choropleth map construction.

Test Map Accuracy

The overview accuracy index was applied to the test maps and three class generalization, as another means of setting standards for response analysis. The results of this application are found in Figure IV.9, in which the relationship between accuracy and quantization is displayed. A list of values is found in Table 2, Appendix C. Two major patterns are evident in the graph. The first pattern is that the accuracy

Figure IV.9


OVERVIEW ACCURACY INDEX FOR TEST MAPS
OAI
CLASSES
0.000
0.250
0.500
0.750
1.000
0
25
50
75
100
125
150
175
200

level is high for all seven maps, and covers only a small range of the accuracy index, from .85 for the 3 class generalization to 1 for the unquantized map. This situation is caused by two major factors. The first factor is the nature of the original distribution used to construct the six test maps. The distribution is linear, with no apparent natural breaks. The optimum generalization technique was very well suited to the data structure, providing data classification without any serious alterations of the structure, other than the presence of classification steps. The second factor is the utilization of a common program that was designed to maximize the overview accuracy of the generalizations at each level. That is, the 3 class generalization created using the program would possess the highest overview accuracy index possible for a 3 class generalization of this data set. The same statement could be made regarding the other six test map generalizations. A combination of these factors resulted in the observed accuracy pattern, setting a narrow standard of variation between test maps which would also strongly affect the observed range of variation between response maps. The visual pattern of the original data was the chief consideration in choosing the data set from which the test maps were constructed; therefore, the linear statistical distribution affecting the range and level of test map accuracy in this study was not anticipated. The standards set by these maps must be accepted for this study; however, desirability and possibility of obtaining both statistical and visual variation in mapped pattern for future studies is recognized, and will be discussed further in the conclusions of this study.

The second pattern featured on the graph is very similar to the pattern seen for the complexity/contrast index. There is a steep rise from the 3 class up to the 7 class generalization, then a gradual rise to the 20 class generalization, and finally a complete leveling off to the 183 class map. Once again, the considerable degree of consistency between the original data distribution and generalized distributions is apparent. This situation is particularly evident beyond the 20 class generalization level, where addition of information does not appear to have any effect on map accuracy.

Conclusion

These three aspects of the original data were examined and displayed in graphs in order to provide a standard for evaluation of viewer response. The standards will be referred to, particularly, in the discussion of patterns and relationships between subsamples, because the data are represented according to test map quantization level. Thus, the same axes are used in the construction of graphs, and visual comparisons can be made easily between patterns in the original data and patterns in viewer response.

Analysis of Relationships Within Subsamples

Introduction

The analysis undertaken within subsamples as conducted for the chief purpose of establishing whether or not there was enough agreement in response within subsamples to allow the median response maps to be used as representatives of the subsamples. Unity of patterns within subsamples would allow comparison of patterns between subsamples, using median response maps as a standard for each test map.

The measures of the level of representation of individual responses by a median response are based, essentially, upon agreement level in response error. This approach has been taken due to the problems associated with the handling of two dimensional responses discussed earlier. The response accuracy and unity measures provide a single value for each response or unit, respectively, which can be compared to a mean value for each subsample. It must be accepted, for the purpose of this study, that response distribution is measured in this fashion, and that the results of this measurement permit use of the median response maps as valid representations of the six response subsamples.

Response Unity

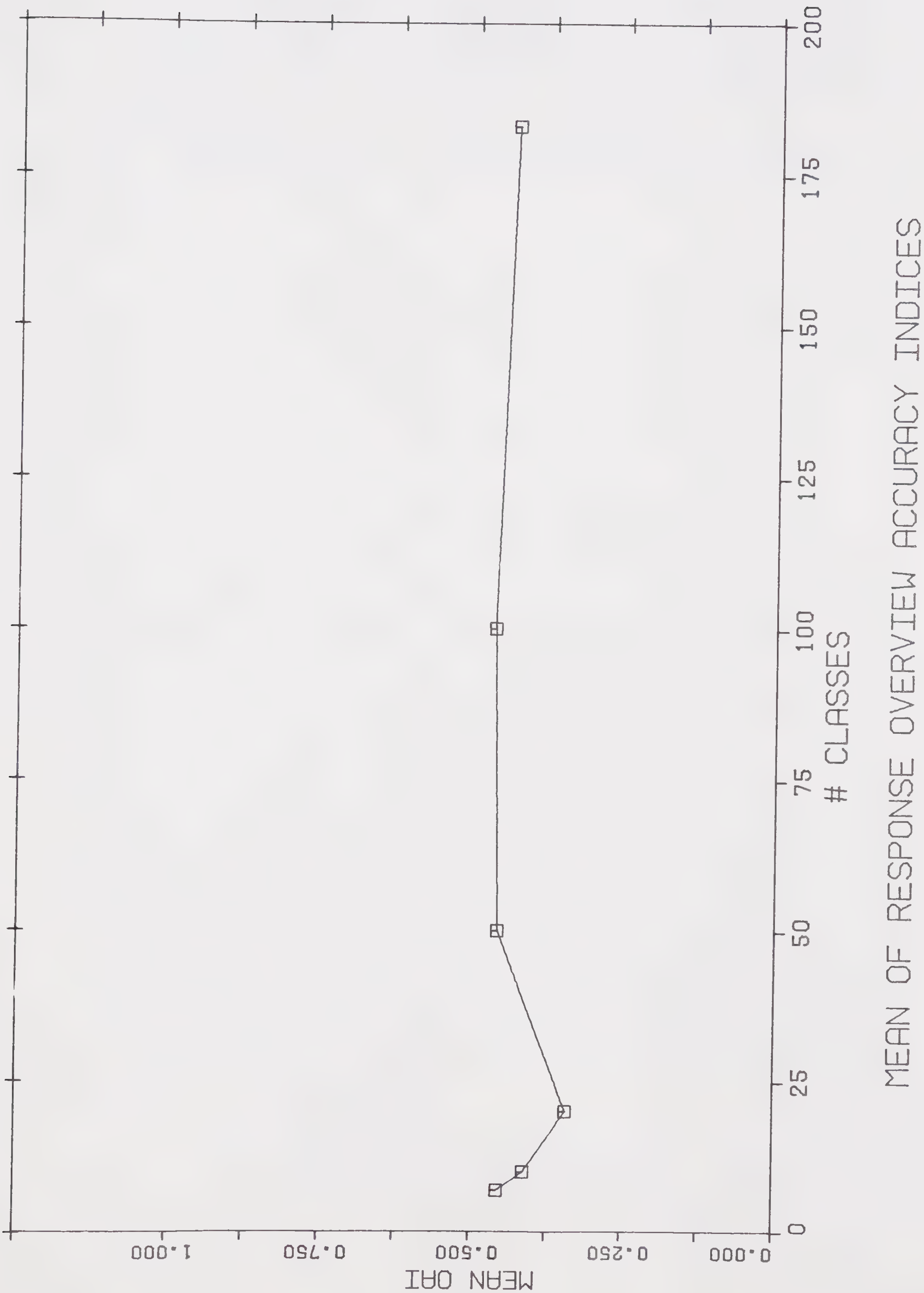
The first measurement of response unity was the calculation of mean deviation from median response values for each set of thirty responses, within subsamples. It was the purpose of this procedure to establish the unity of response deviation within each subsample. The calculations were made using the tables of deviations from median response located in Appendix C. The individual number of

deviations from the median response were calculated for each unit on the map for each subsample. The mean deviation level and the standard deviation were then calculated for each subsample of response deviations. It was discovered, for all six subsamples, that the individual totals of deviations from median responses for each unit on the test map were normally distributed about the mean deviation value for that subsample. Thus, evidence is provided that the median response maps are good representations of the individual response maps. The mean deviation values obtained for each subsample will be discussed in the evaluation between subsamples.

Response Accuracy

The response accuracy was calculated for each of the 180 response maps using the overview accuracy index procedure. Then, the mean and standard deviation were calculated for each set of 30 overview accuracy indices per subsample. The individual overview accuracy indices were found to be normally distributed about the mean of the overview acccuracy indices for each subsample. The accuracy levels were low, since the class limits set for the response maps were based on extremes in response values for each class.(see Figure IV.10) Actual values are listed in Table 3, Appendix C. (note: these values are listed according to test group; any other ordering in the listing is arbitrary, since a different respondent was used for each of the 180 maps tested) The accuracy index did not take into account the fact that some class limits were very high, while others were very low. It was expected that these patterns could be used to conduct an analysis of variance of response accuracy which would provide a statistical measure of patterns within and between subsamples. However, evidence of response extremes in all subsamples indicated that reduction in number of classes had no apparent effect on the presence of extreme errors in response; thus, it was concluded that analysis of variance would not lead to any further explanation of variation between subsamples. The median maps used to evaluate viewer response represent a smoothing of individual response maps, with the elimination of extremes that are not part of the general overall response.

Figure IV.10


MEAN OF RESPONSE OVERVIEW ACCURACY INDICES
CLASSES
MEAN OAI
0.000
0.250
0.500
0.750
1.000
0
25
50
75
100
125
150
175
200

Conclusion

It may be concluded from the results of these two statistical tests that the median response maps are useful and valid representations of the thirty original response maps collected for each test map. The six median response will be used to evaluate and display variations between the six subsamples.

Analysis of Relationships Between Subsamples and Comparisons with Original Data and Test Maps

Introduction

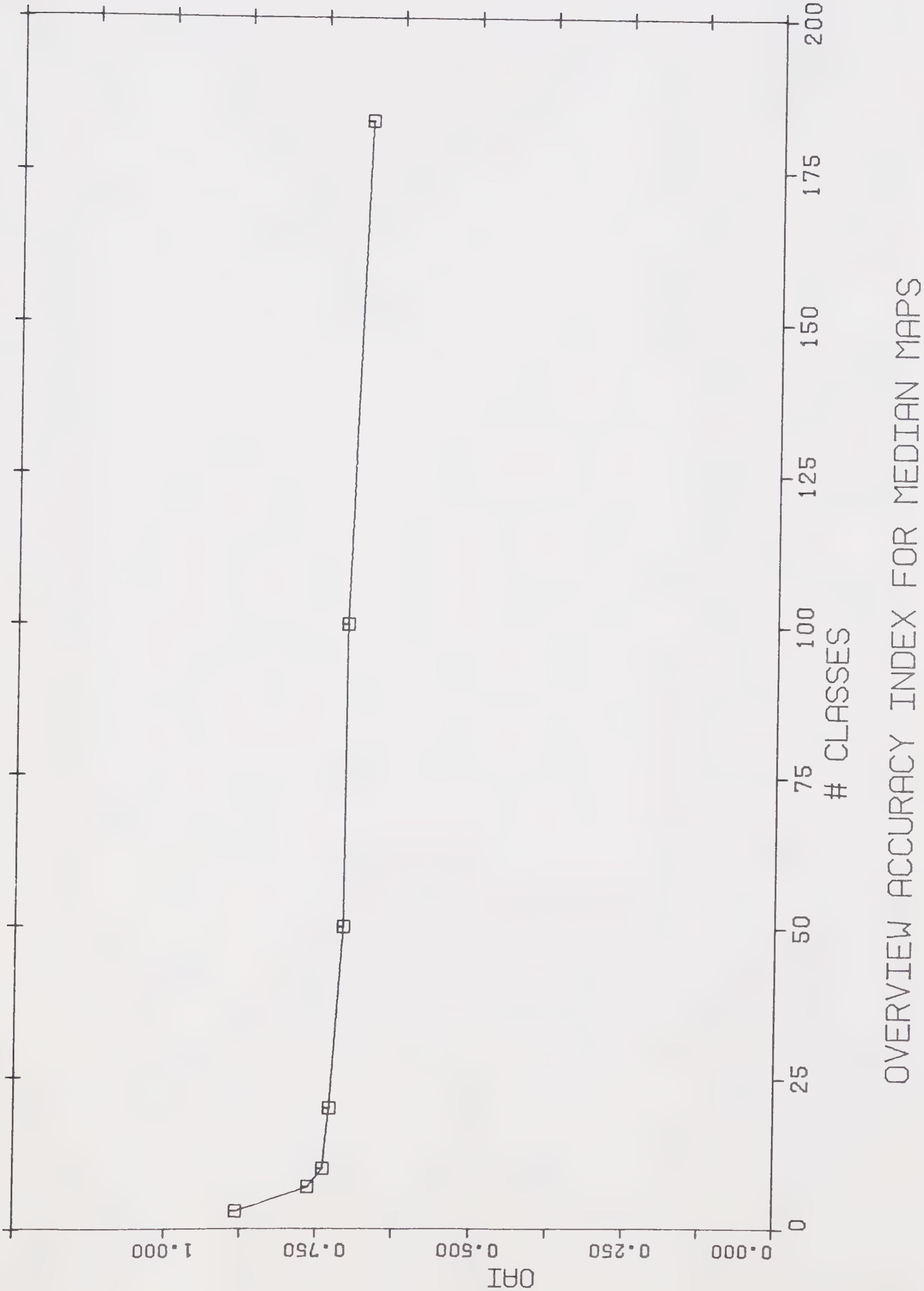
The median response maps for each subsample were evaluated according to accuracy, consistency, and unity of response. Results were graphed by unit number and test map number. Thus, patterns were viewed as trends varying within quantization level of test map, and between quantization levels of test maps.

Response Accuracy

Response accuracy of the three class median response maps was represented by the overview accuracy index. The pattern of response accuracy varying with test map quantization level is displayed in Figure IV.11. A list of values is presented in Table 4, Appendix C. The line between the 3 and 7 class level indicates the expected drop from the maximum possible accuracy for a three class map, the value of the overview accuracy index for the 3 class optimum generalization. In making this graph, it was assumed that response to a 3 class test map would be completely accurate for all respondents. The pattern displayed on this graph represents an interesting reversal of the pattern displayed in Figure IV.9, which shows the variation in accuracy for test map quantization level. Here, there is a sharp drop down to the 7 class response, then a more gradual slope to the 50 class response, and a very gradual decline to response accuracy for the 183 class, unquantized, map. The range in accuracy is larger than for the original test maps. All accuracies are lower than the lowest accuracy for the optimum generalization, the accuracy of the 3 class optimum map.

The rate of accuracy loss in response decreases as the number of classes increases. This observation is not unusual, when the complexity level and photographic reproduction error of the test maps are taken into account. For this

Figure IV.11

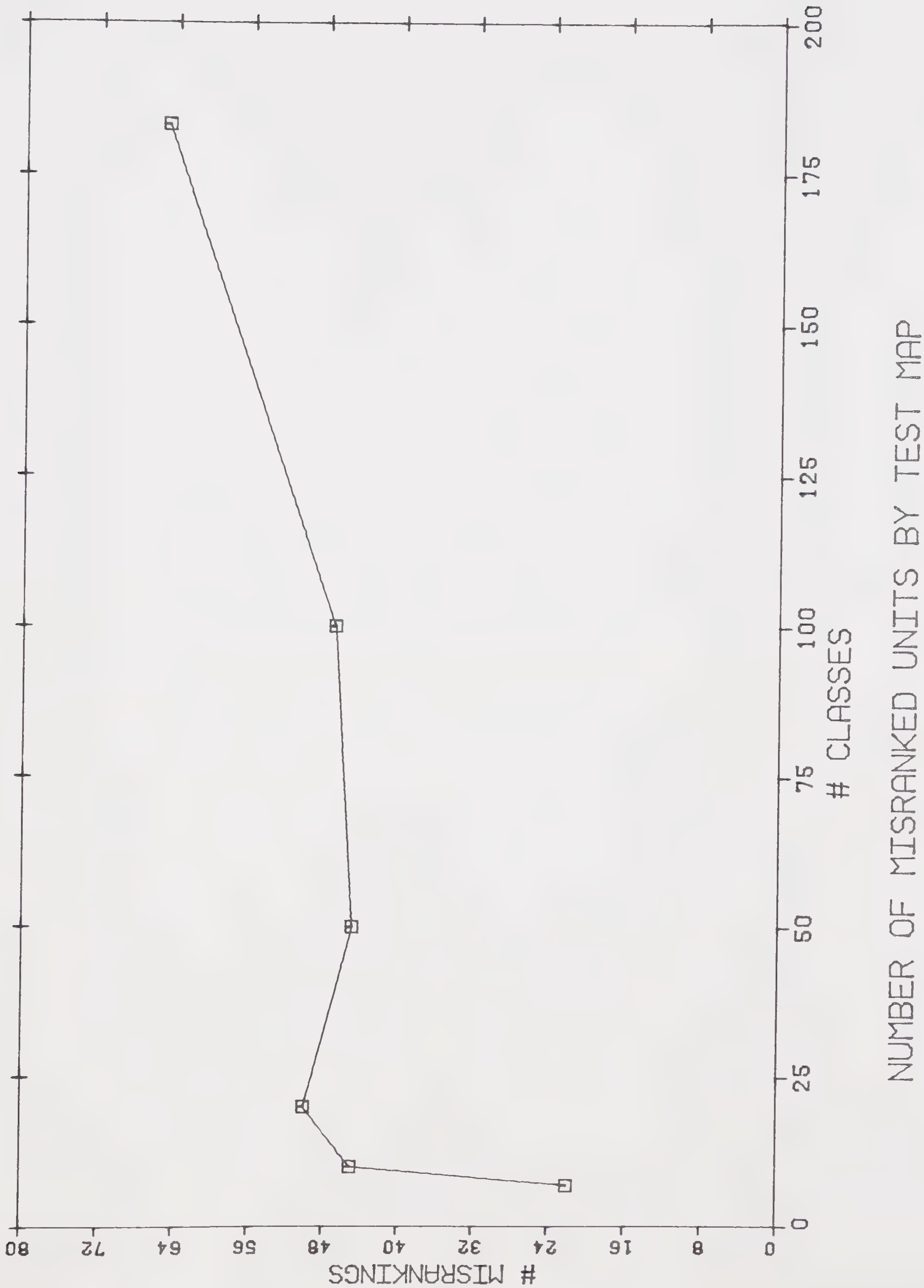

OAI
0.000
0.250
0.500
0.750
1.000
0
25
50
75
100
125
150
175
200
CLASSES
OVERVIEW ACCURACY INDEX FOR MEDIAN MAPS

data set, the complexity index levels off at the 50 class generalization. A similar trend is seen in response accuracy. It is, therefore, possible to produce maps with higher accuracy without increasing complexity, but the usefulness of this practice in the presentation of general patterns of information is questionable: although accuracy of the test maps increases with quantization level, differences in information presented and obtained may be negligible. The effects of photographic reproduction on the visual appearance of the test maps must also be considered as a factor which might influence viewer response, particularly response to the less generalized maps in which upper density levels might be generalized by the reproduction method used. Those areas of the map automatically classified by reproduction noise would be automatically classified by the map viewer as well. In this project, the complexity values and range were narrow for the test maps. Thus it may be expected that reproduction noise would have limited effect on the visual appearance of the mapped pattern, and thereby limited effect on response patterns. However, if complexity range and values were high, then photographic reproduction noise could be a significant factor in limiting the visual impact of this complexity, as perceived by the map viewer. Owing to low variation in accuracy and complexity in original data distribution and test maps, it will be assumed that generalization of data inherent in photographic reproduction of test maps had little effect on observed response patterns. The ramifications of these observations in the field of choropleth mapping will be discussed in the concluding chapter.

Response Consistency

The measure of response consistency concerns the presence of misranked units in median response maps. The total number of misranked units for each of the six subsamples was graphed according to test map quantization level, as seen in Figure IV.12. These values, along with values for each ranking class are found in Table 5, Appendix C. The general pattern consists of a sharp upward trend to the 20 class response, with a shallower angle rising to the 183 class response. In a detailed view of the ranking errors in response maps, a slight dip in the error for the 50 class response is seen. A possible explanation for this dip is not readily apparent.

Figure IV.12


NUMBER OF MISRANKED UNITS BY TEST MAP
CLASSES
MISRANKINGS
0
25
50
75
100
125
150
175
200
0
8
16
24
32
40
48
56
64
72
80

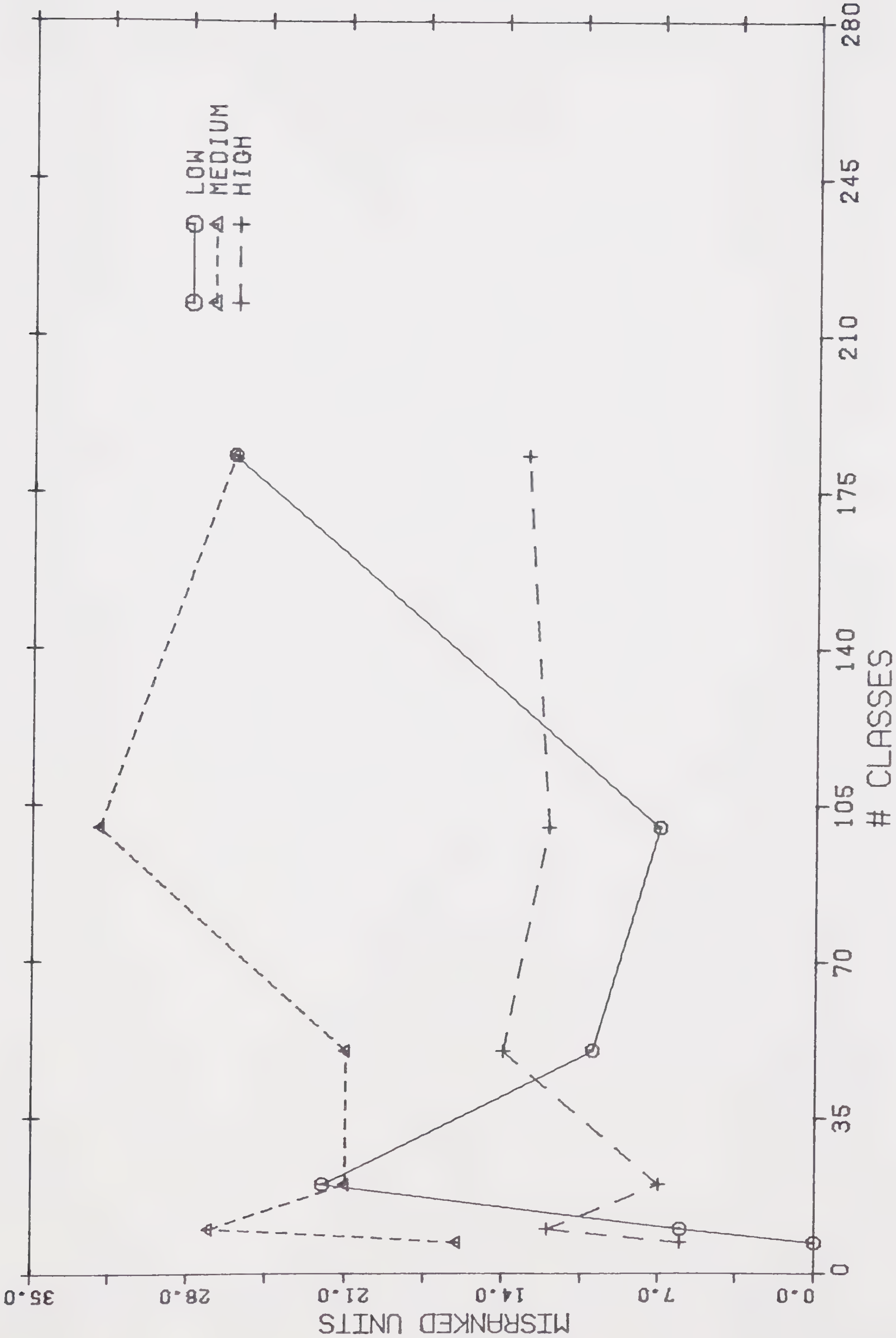
The number of misranked units was also graphed according to the three classes of response, high(3), medium(2), and low(1), as displayed in Figure IV.13. These values are calculated according to the total number of misranked units within the class limits of each of the three classes for each median response map. The graph shows the considerably higher level of ranking error in the second class, presumably due to the fact that this class incorporates both overestimation of lower values and underestimation of higher values in the range between the highest and lowest values. This observation is later strengthened by examination of error distribution over the mapped surface.

The ranking distributions were also graphed, to show deviations from the original data distribution ranking, shown in Figures IV.14–IV.19. The graphs display disruptions created by misranked units in the median maps. The 7 class response distribution shows the lowest alteration to the original distribution, with no change to data ranking between the first and second class. The level of ranking error alters the pattern of the distribution most significantly in the 183 class response. The other response groups possess an increasing level of error, with varying ranking problems between the first and second classes or second and third classes.

Response Unity

The final measure of response was the comparison of individual response agreement with median responses between the six subsamples. The results of mean number of deviations from median values versus test map quantization level are exhibited in Figure IV.20. The mean values and standard deviation for each subsample are listed in Table 6, Appendix C. Again, there is a sharp rise in deviation at the lower level, a repetition of the unexplained dip at the 50 class level, and a leveling at the higher levels. The pattern of this graph closely approximates the trends exhibited by the graph of ranking errors in median responses. Graphs of deviations according to unit density present evidence of possible sources of response error. The deviations from median response according to unit are found in Figures IV.21–IV.26. The greatest number of errors occur in the middle areas of the data, those areas which represent a transition between the darkest and lightest density areas. The graphs of deviations from median response according to unit

Figure IV.13



NUMBER OF MISRANKED UNITS BY CLASS

Figure IV.14

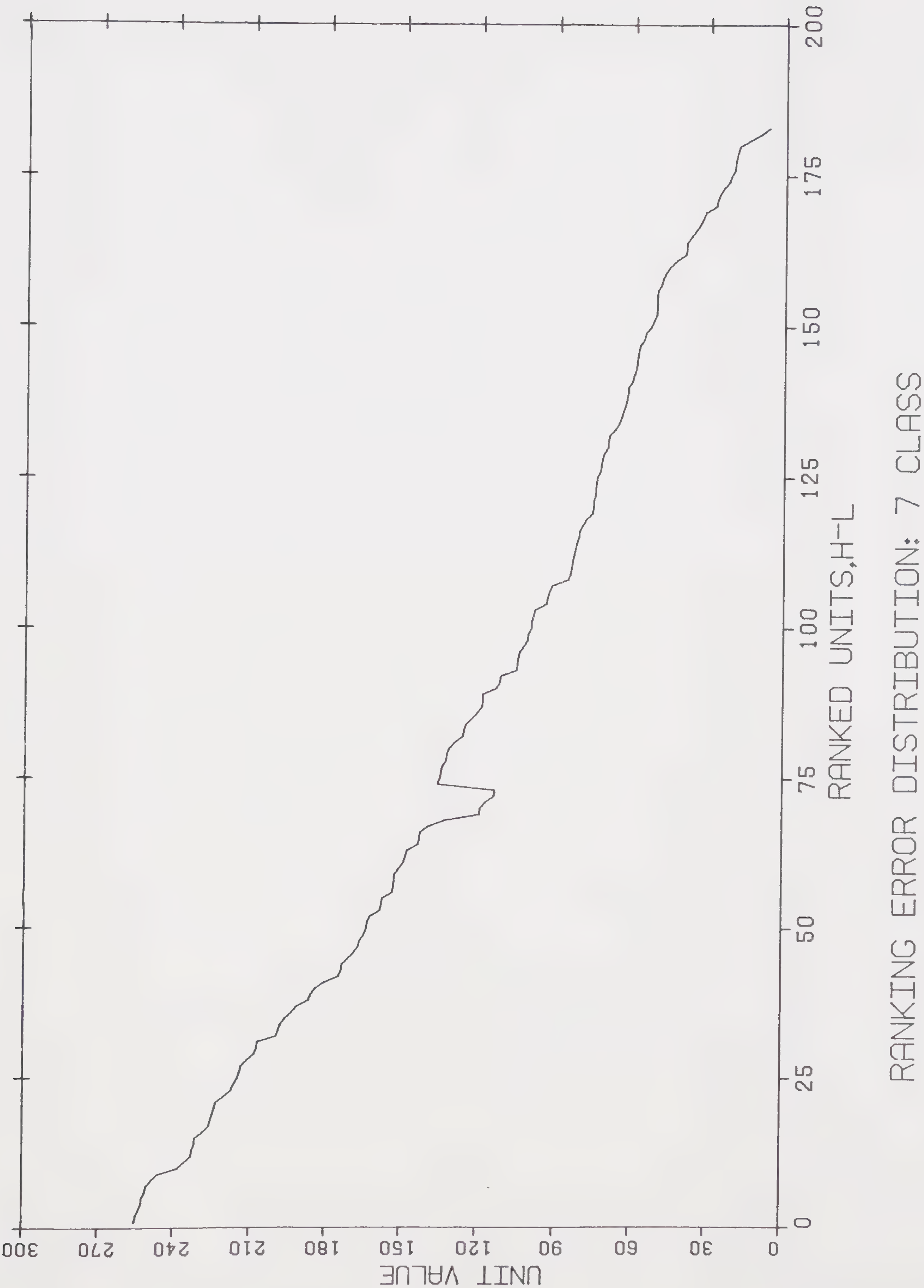

RANKING ERROR DISTRIBUTION: 7 CLASS
RANKED UNITS,H-L
UNIT VALUE
0
25
50
75
100
125
150
175
200
0
30
60
90
120
150
180
210
240
270
300

Figure IV.15

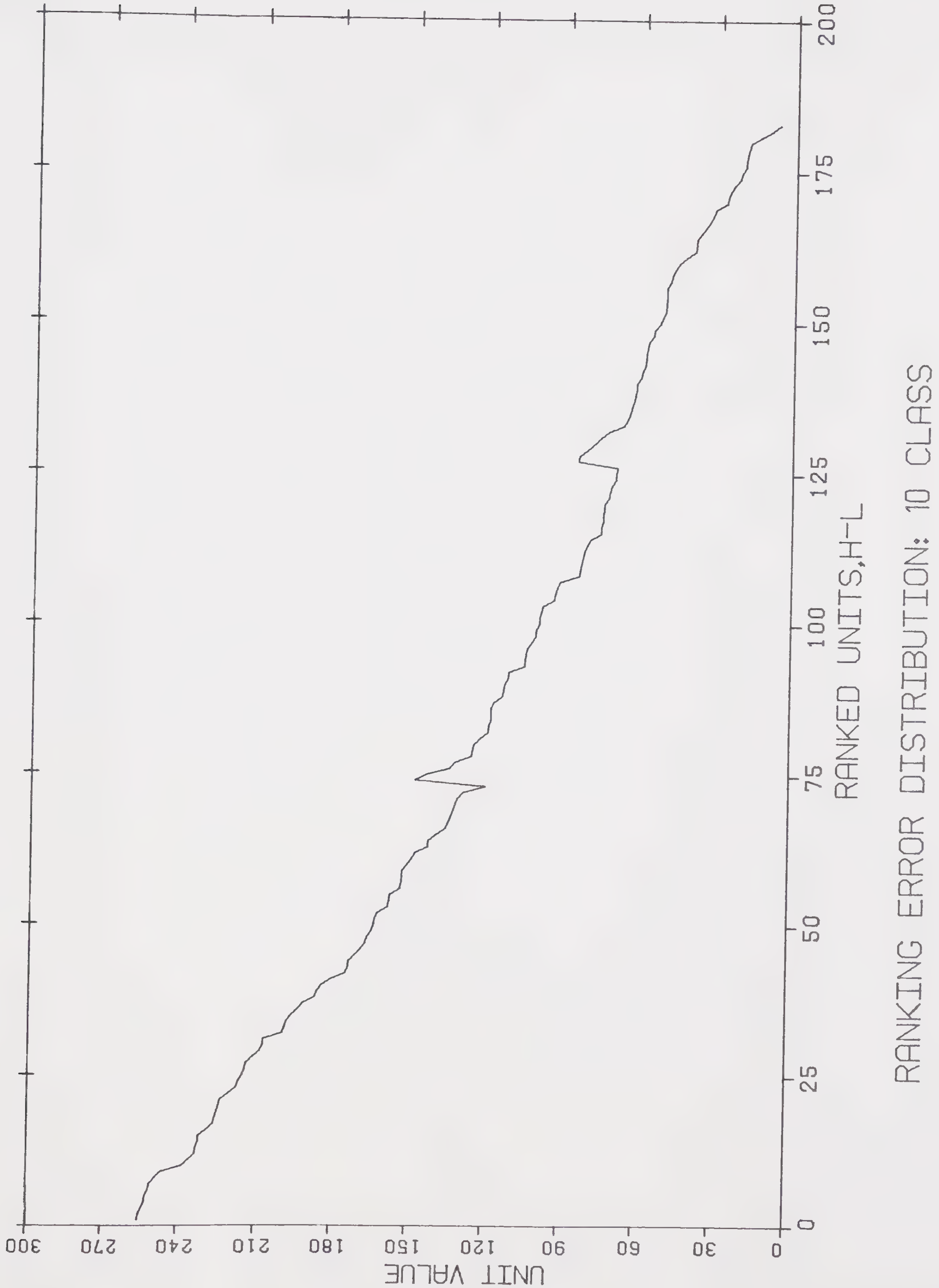

RANKING ERROR DISTRIBUTION: 10 CLASS
RANKED UNITS,H-L
UNIT VALUE
0
25
50
75
100
125
150
175
200
0
30
60
90
120
150
180
210
240
270
300

Figure IV.16

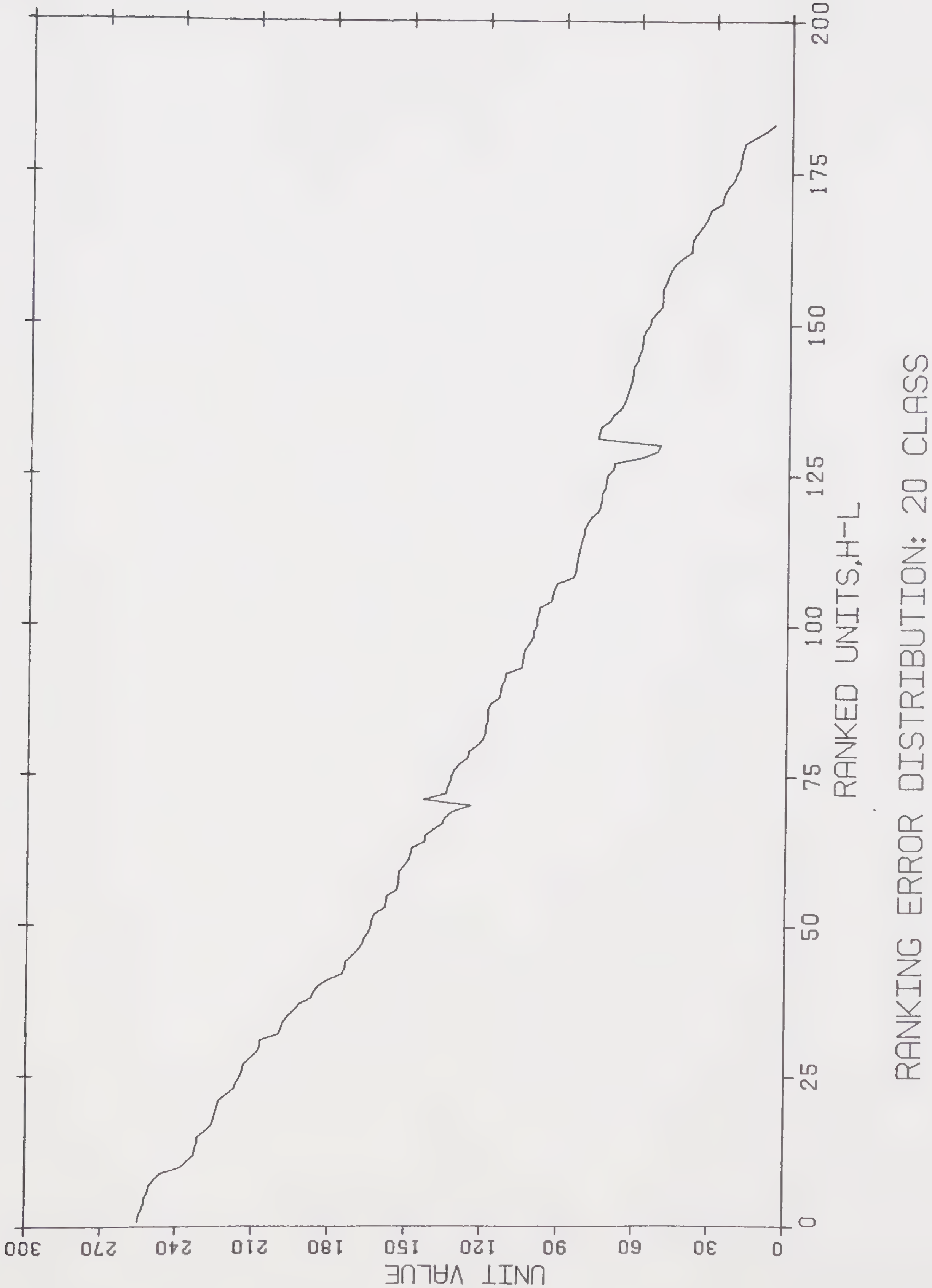

RANKING ERROR DISTRIBUTION: 20 CLASS
RANKED UNITS,H-L
UNIT VALUE
0
25
50
75
100
125
150
175
200
0
30
60
90
120
150
180
210
240
270
300

Figure IV.17

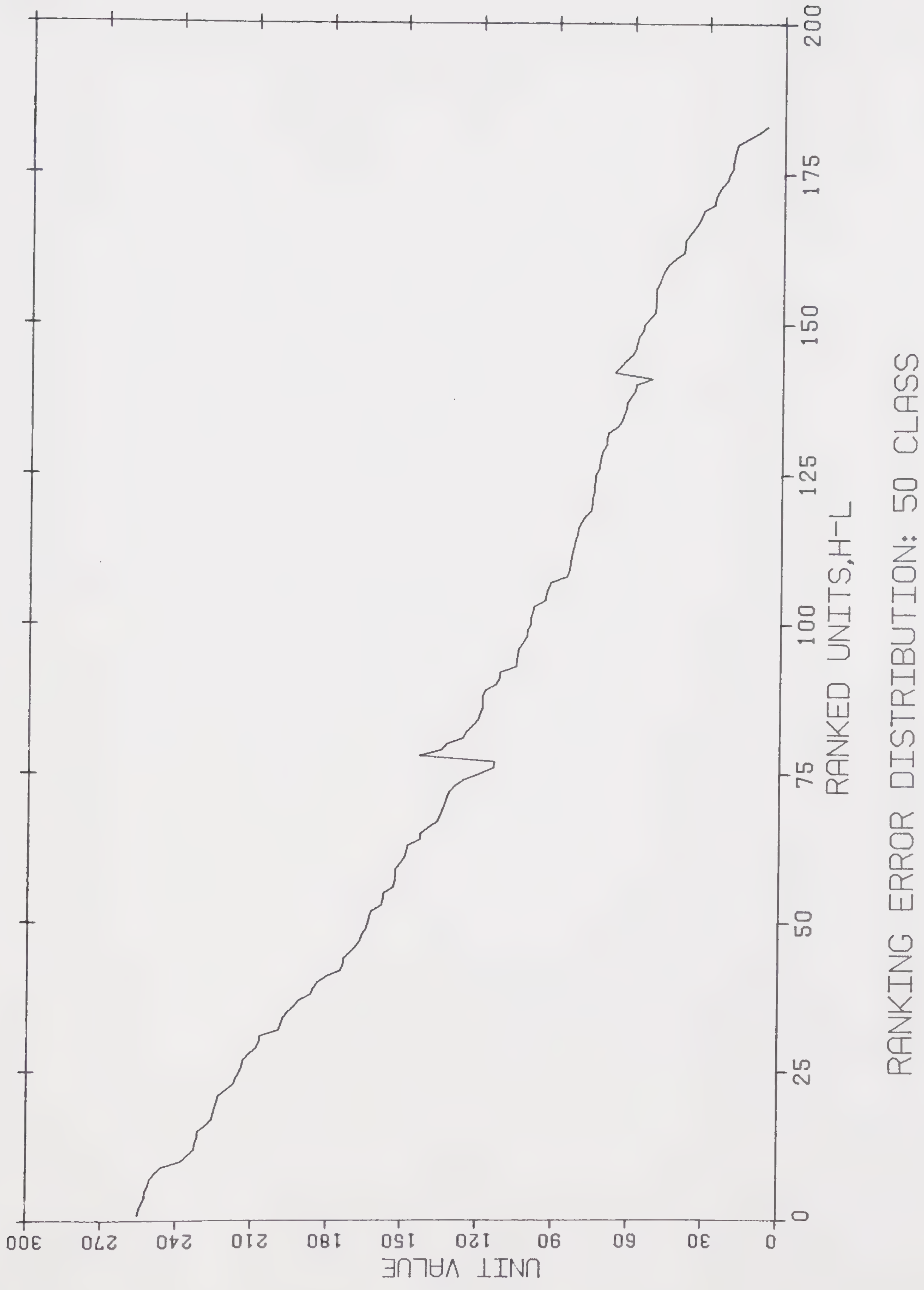

RANKING ERROR DISTRIBUTION: 50 CLASS
RANKED UNITS,H-L
UNIT VALUE
0
25
50
75
100
125
150
175
200
0
30
60
90
120
150
180
210
240
270
300

Figure IV.18

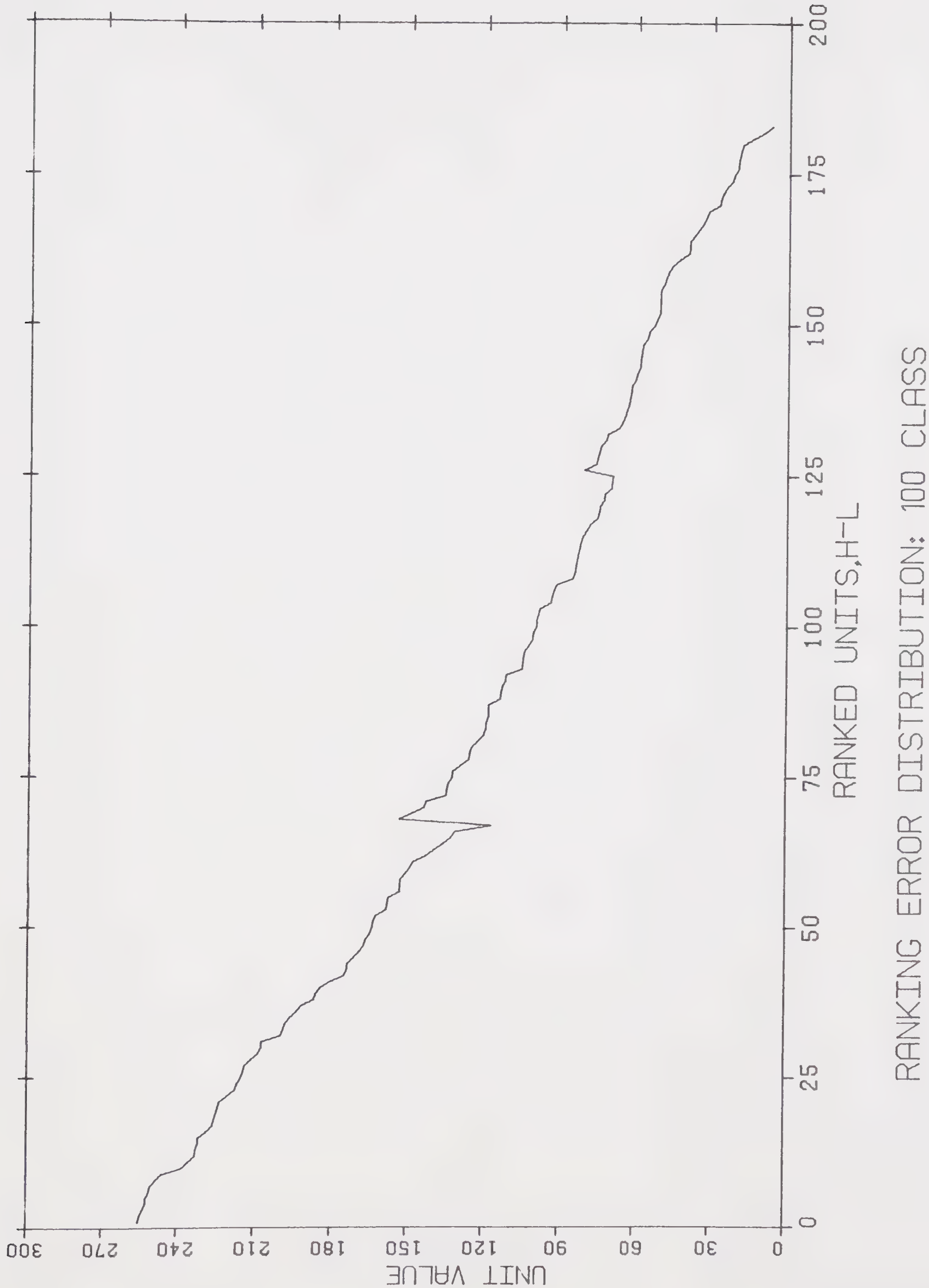

RANKING ERROR DISTRIBUTION: 100 CLASS
RANKED UNITS,H-L
UNIT VALUE
0
25
50
75
100
125
150
175
200
0
30
60
90
120
150
180
210
240
270
300

Figure IV.19

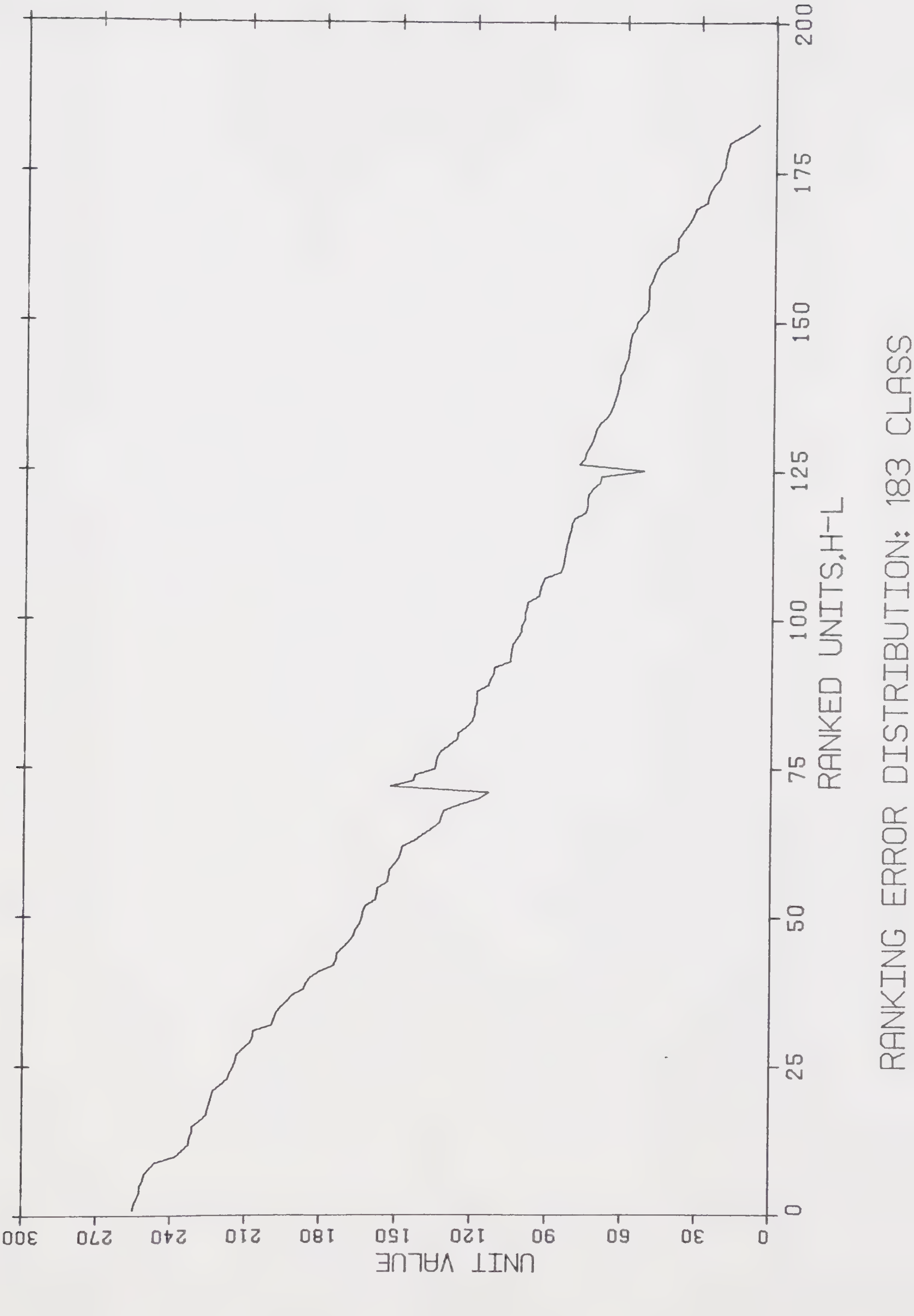
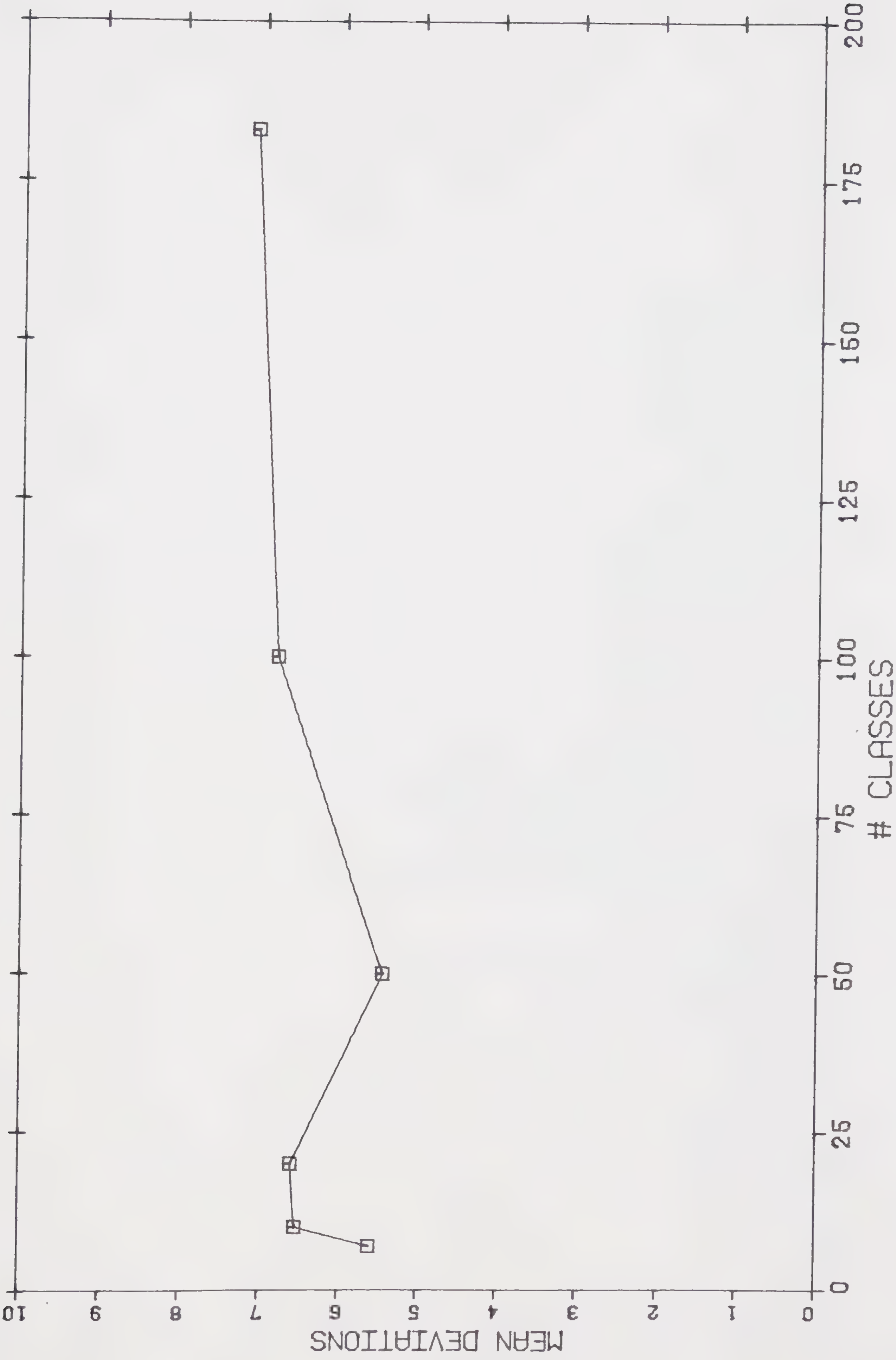

RANKING ERROR DISTRIBUTION: 183 CLASS
RANKED UNITS,H-L
UNIT VALUE
0 25 50 75 100 125 150 175 200
0 30 60 90 120 150 180 210 240 270 300

Figure IV.20



MEAN RESPONSE DEVIATIONS BY TEST MAP

Figure IV.21

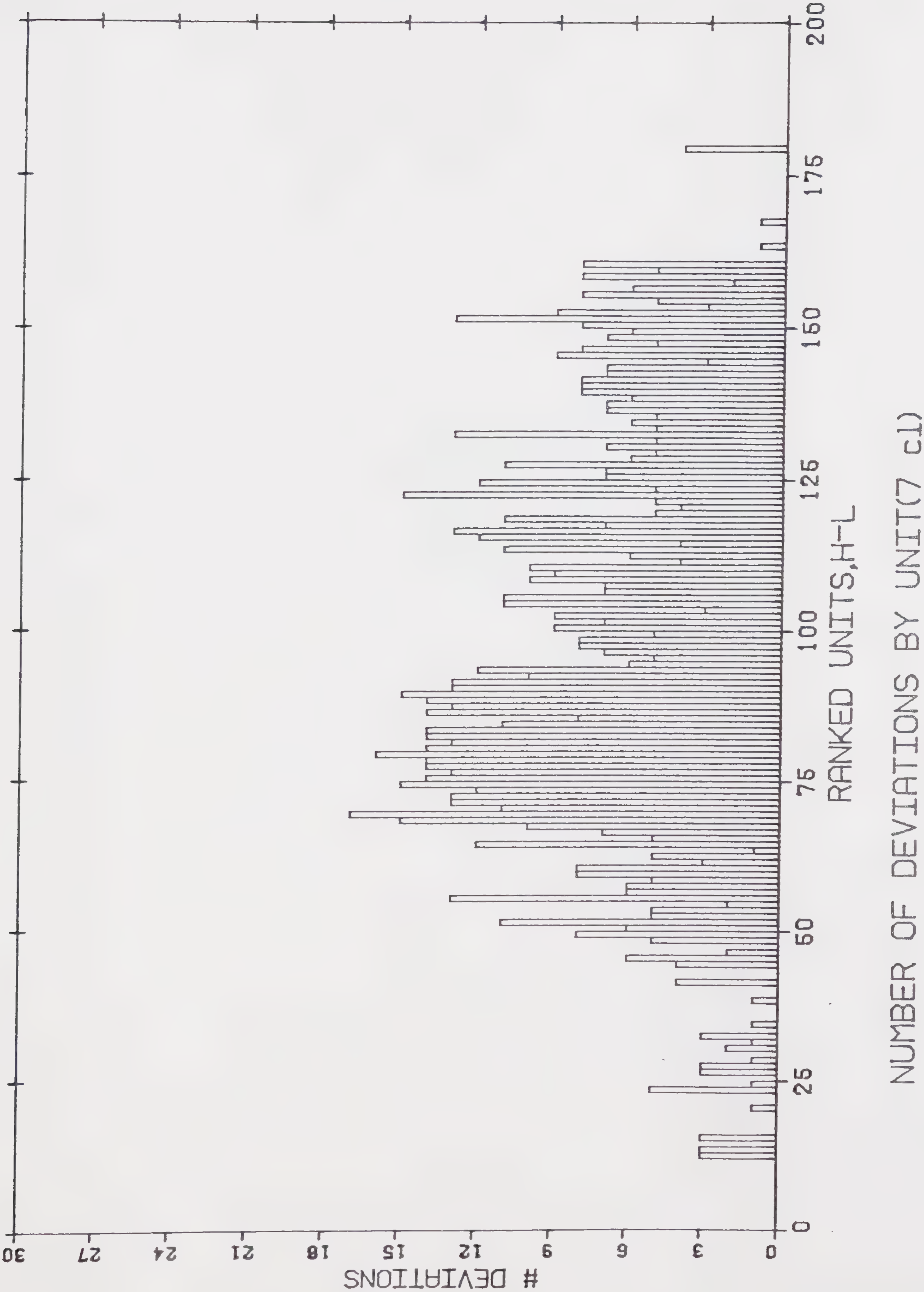
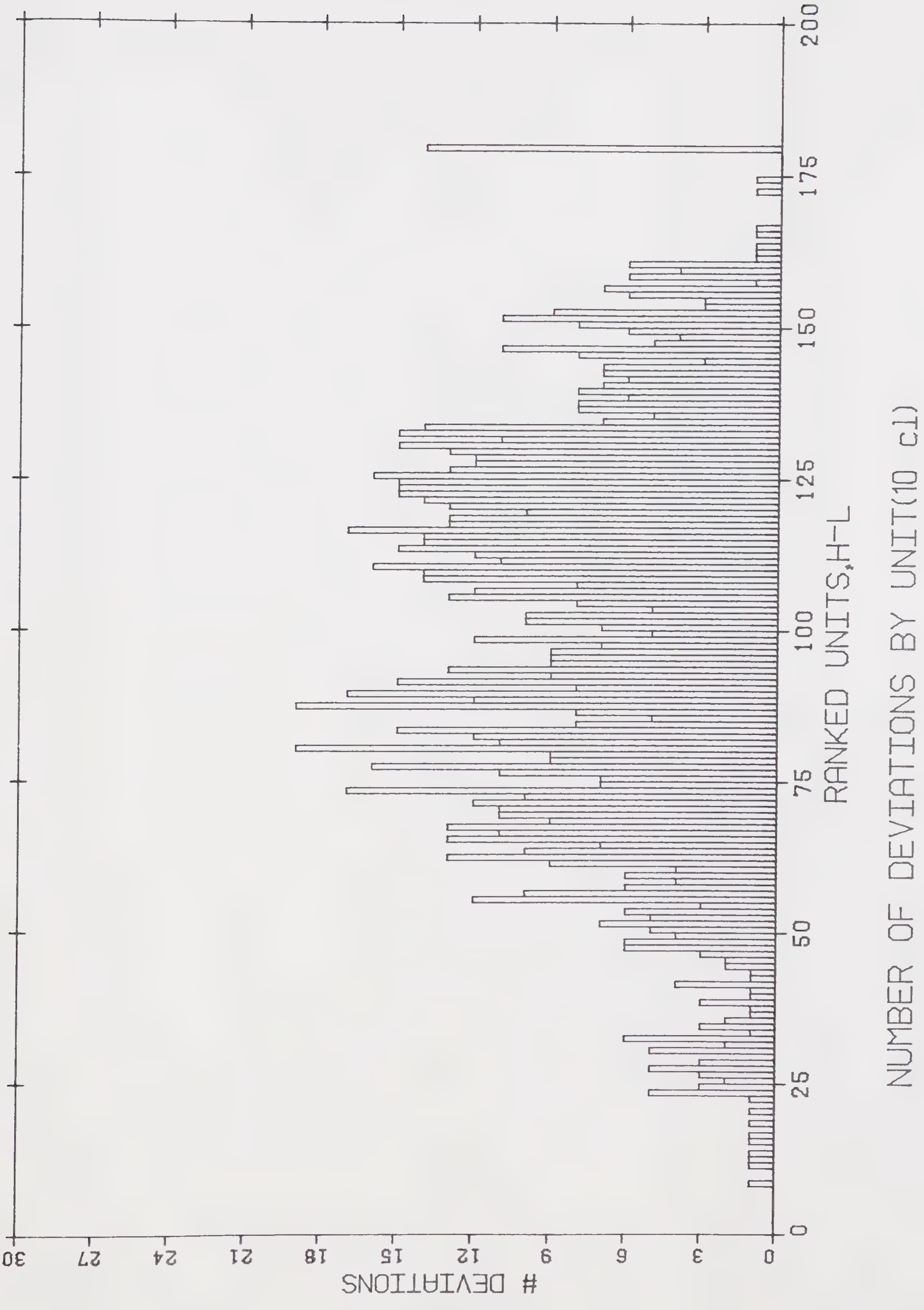

NUMBER OF DEVIATIONS BY UNIT(7 cl)
RANKED UNITS,H-L
DEVIATIONS
0
25
50
75
100
125
150
175
200
0
3
6
9
12
15
18
21
24
27
30

Figure IV.22



DEVIATIONS
0
3
6
9
12
15
18
21
24
27
30
0
25
50
75
100
125
150
175
200
RANKED UNITS,H-L
NUMBER OF DEVIATIONS BY UNIT(10 cl)

Figure IV.23

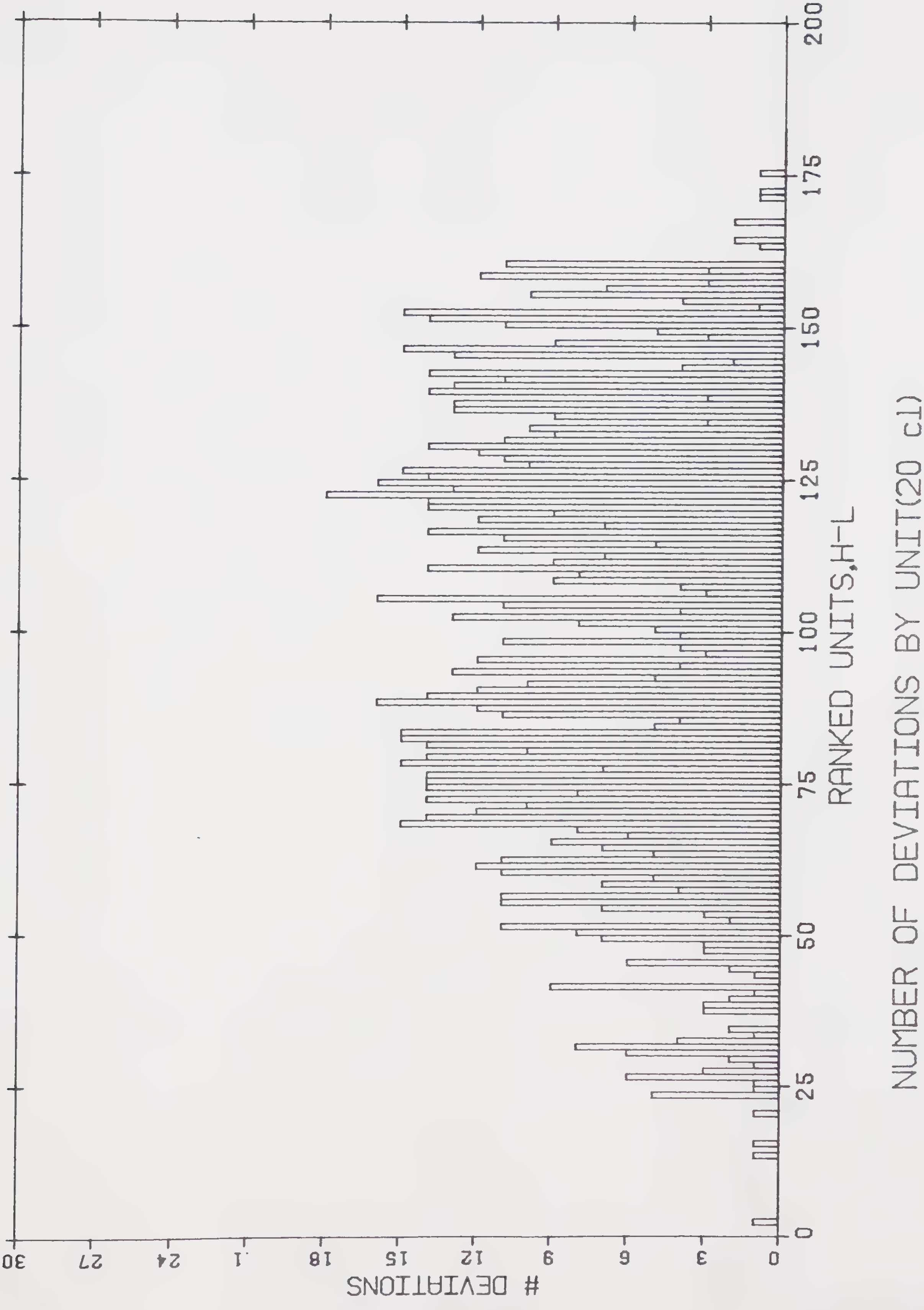

NUMBER OF DEVIATIONS BY UNIT(20 cl)
RANKED UNITS,H-L
DEVIATIONS
0
3
6
9
12
15
18
21
24
27
30
0
25
50
75
100
125
150
175
200

Figure IV.24

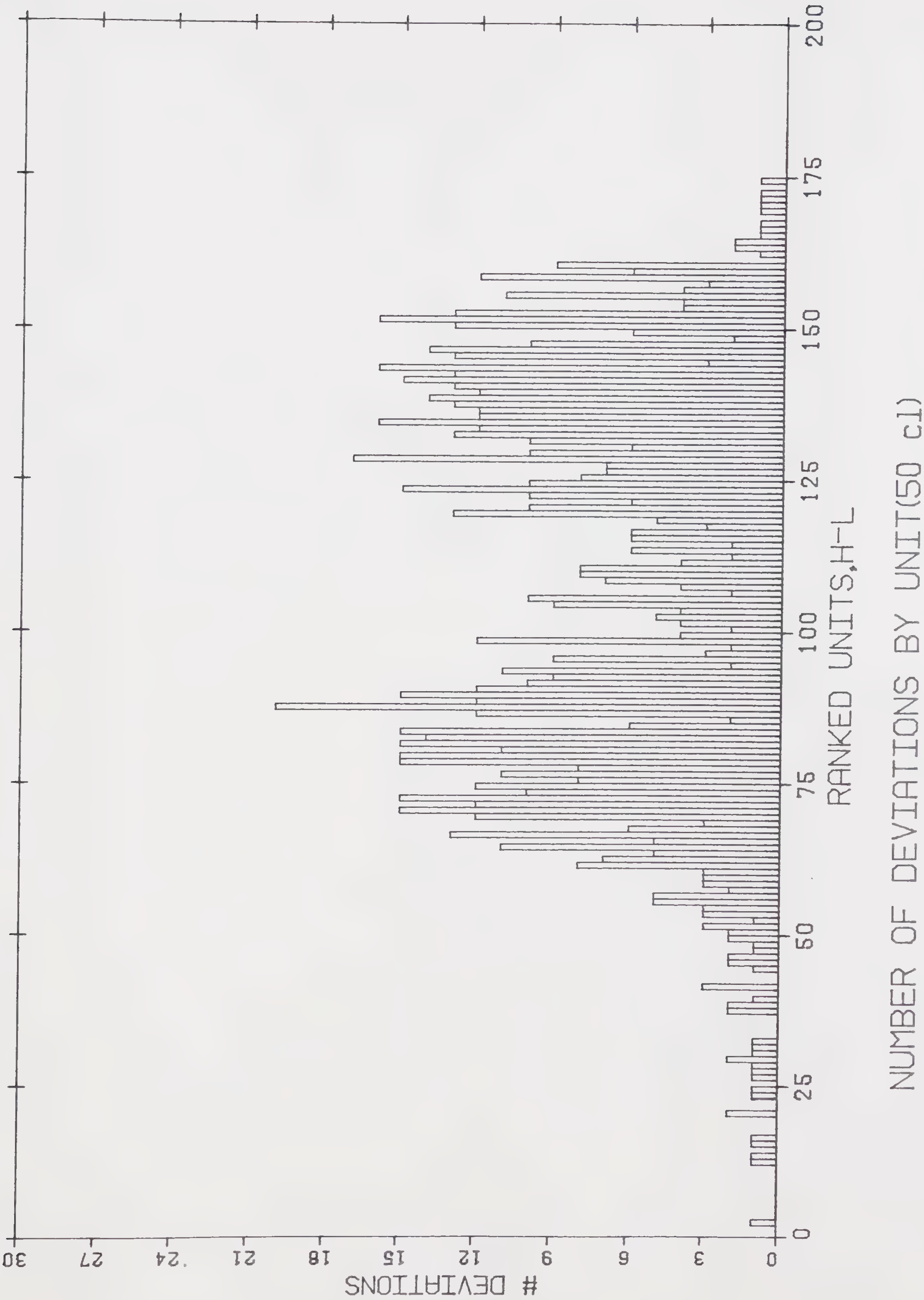
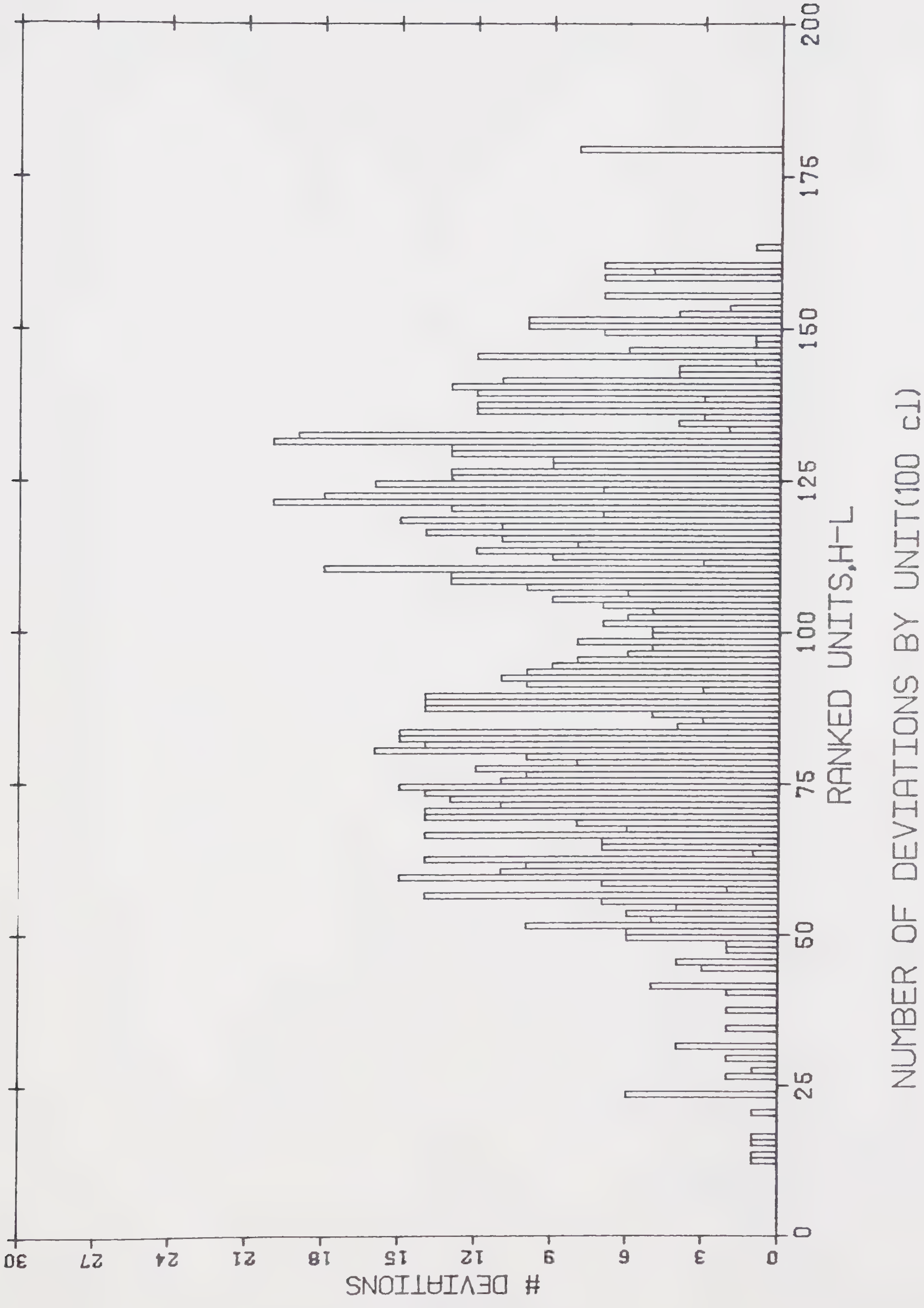
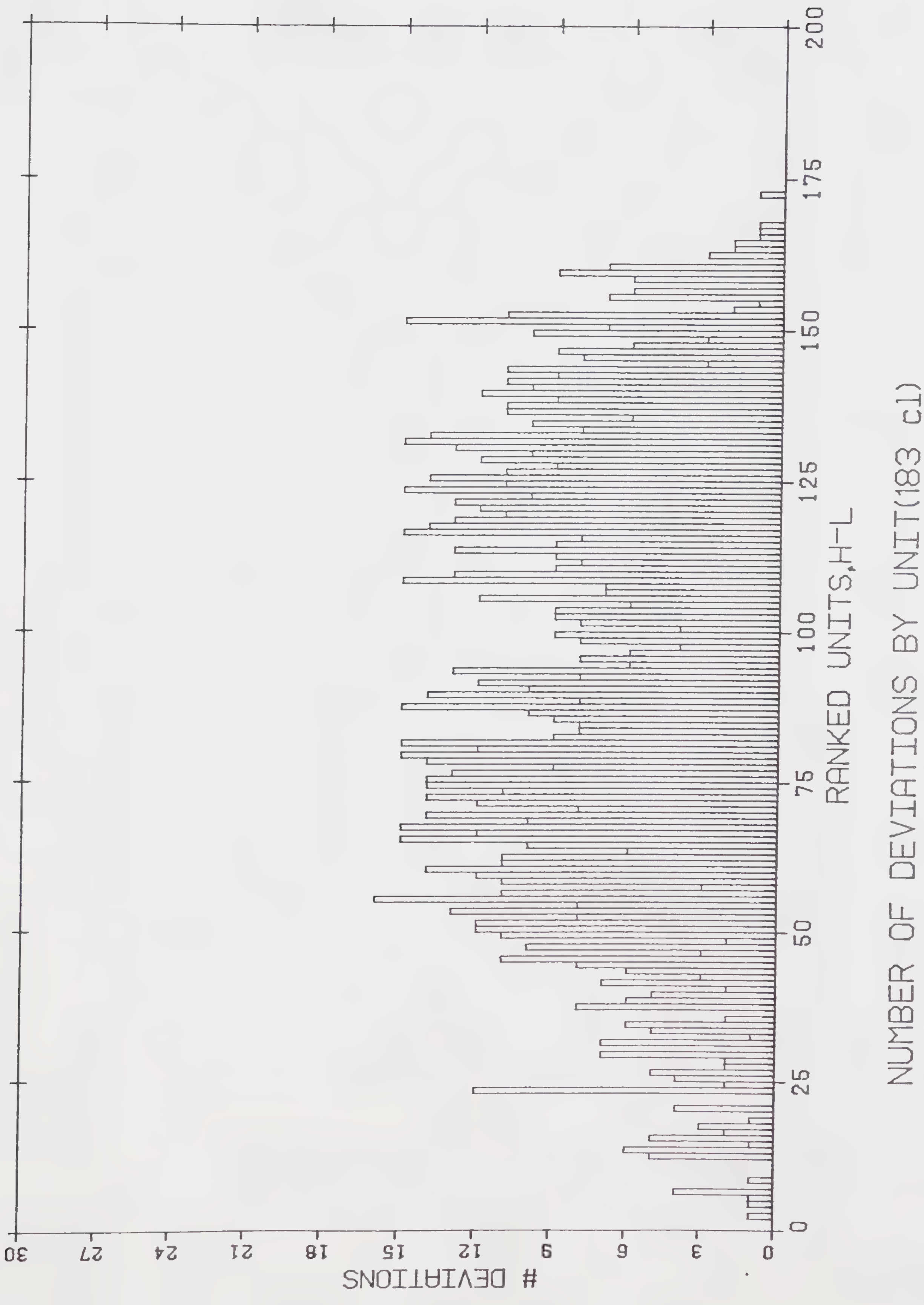

NUMBER OF DEVIATIONS BY UNIT(50 cl)
RANKED UNITS,H-L
DEVIATIONS
0
25
50
75
100
125
150
175
200
0
3
6
9
12
15
18
21
24
27
30

Figure IV.25



NUMBER OF DEVIATIONS BY UNIT(100 cl)
RANKED UNITS,H-L
DEVIATIONS
0
25
50
75
100
125
150
175
200
0
3
6
9
12
15
18
21
24
27
30

Figure 1V.26


NUMBER OF DEVIATIONS BY UNIT(183 cl)
RANKED UNITS,H-L
DEVIATIONS
0
25
50
75
100
125
150
175
200
0
3
6
9
12
15
18
21
24
27
30

density present some interesting patterns. The 7 class deviation graph shows a lower level of disagreement from the median response, with peaks in the middle classes. Over the rest of the test maps, these peaks increase in number and height with quantization level until the 183 class map, which shows a spreading of the peaks to the extreme end of the distribution. The 50 class deviation graph shows a pattern which further defines the observed anomaly in the ranking error and mean deviation graphs. The peaks are high, but there is a low amount of deviation at both ends and a considerable drop in deviation in the center of the distribution. The pattern of lower deviation in the centre of the distribution is also observable in the other graphs, clearly showing the difficulties in response along the borders between very high and very low values, with a very narrowly defined stable middle class. Patterns on the other five graphs are far less defined. The graph makes clear the pattern of the 50 class response, but does not really explain the factors which might have caused it. In search of a reason for the lower deviation in the 50 class test group, the areal pattern of error, according to map unit, was studied for each test map. No distinct areal patterns of response deviation in the response maps, other than the previously observed problems in transition zones between high and low areas were observed. The complexity/contrast index calculated for the test maps shows a leveling at the 20 class mark. Therefore, the visual contrasts which might affect the perception of areas adjacent to very light or very dark areas should not be a factor here. The only notable difference between the 50 class response maps and the other five sets of response maps is the relatively high standard of neatness on these maps. This may indicate a higher level of thought and care in response to the map, including more attention to "problem" areas on the map, ignored by more careless respondents. The unusual pattern, while raising interesting questions, does not appear to affect significantly the perception of general response patterns. Generally, the transition areas of the map appear to be the chief source of problems in viewer response unity.

Conclusion

Relationships between subsamples have been studied using the measures of response accuracy, consistency, or unity. Results were presented in the form of

graphs, to allow comparison between responses to units on the maps, and between responses for subsamples. This process represents the final stage in data processing, and provides results which can be used in making conclusions about the data.

F. Summary of Results

The basic levels of data processing have been completed, providing the information about patterns and relationships in the data structure from which conclusions will be drawn. Definite, and apparently related, patterns have been seen in response patterns in several forms of the data. From these patterns, conclusions about relationships within the data structure will be made. For quick reference purposes, the results of this analysis will be summarized as follows:

1. Original Distribution and Test Maps
    a. linear distribution of test map data, moderately undisturbed by optimum generalization.
    b. Accuracy
        1) high values.
        2) narrow range.
        3) general pattern of steep rise from more generalized maps to complete leveling at the 50 class generalization.
    c. Complexity
        1) low values.
        2) narrow range.
        3) similar pattern to accuracy, with exception of peak at the 7 class generalization.
2. Viewer Response Maps
    a. level of agreement among individual responses within subsamples.
        1) individual deviations from median response normally distributed about mean deviation from median response for each subsample.
        2) accuracy of individual response maps normally distributed about mean accuracy of response maps for each subsample.

b. Patterns Between Subsamples

1) Unity– increasing disagreement level from highly quantized to unquantized test maps, with exception of dip at the 50 class generalization; problems in transition areas.

2) Consistency– number of misrankings increases in pattern similar to unity in disagreement, including dip at 50 class; most difficulty seen in transition areas between high and medium, and low and medium; increasing deformation of original distribution ranking from 7 class to 183 class generalizations.

3) Accuracy– small range; reverse pattern of test map accuracy, with steep downward trend from 3 class generalization and leveling at the 50 class generalization.

Two unanticipated factors influenced the pattern of results. First, the nature of response maps precluded the use of standard statistical tests. Second, the statistical pattern of the original distribution created a narrow range of accuracy and complexity in the test maps, which set a standard for the narrow ranges in response variation. Despite the effects of these factors, distribution patterns in viewer response were evident. These patterns, combined with the unexpected problems encountered in this study, will provide the source of results from which some of the conclusions about this project will be drawn.

V. Conclusions

A. Introduction

The conclusions set forth in this study have several sources, as well as several focii. The sources of the conclusions comprise the following: the problems encountered in carrying out the study and data analysis, results of the study, and results of previous research in related studies in choropleth mapping. The specialized focii of these conclusions include the following areas: the ramifications of observed problems in the study, the map viewer, the proposed hypothesis, and the quantization controversy in choropleth mapping. A broader focus of the conclusions is defined as the utilization of all that has been learned in this study to obtain a greater understanding of the role of cartographic communication in choropleth mapping, and the contribution of studies in cartographic communication to the development of cartographic theory.

B. Problems Encountered and Tackled in this Project

Introduction

The problems experienced in this project are a very important component of the conclusions which will be presented: these problems have influenced the analysis, results, and overall direction of this research; the basic material from which conclusions are formed. As with any experimental project, unforeseen problems arose at several stages in the study. Some of these problems were resolved through changes made to the procedure or goals of the research. Others presented a more difficult challenge; recognizable, though hardly removable at the particular phase of this research in which they were encountered.

Test Maps

The first problem in this study was the selection of test maps. Owing to the time consuming nature of transferring the data to workable form, the number of subsamples was limited to six. Although 180 individual responses were collected, only six median response maps could be calculated for the sample. This problem was tackled through careful selection of a test map range, coupled with recognition of the limitations to predictive analysis imposed by this range.

After the test maps had been selected and constructed, a second major problem was encountered. The photographic method used to reproduce the original facsimile maps for testing purposes resulted in distortion of the grey scale seen on the originals. With considerations of cost and consistency, the maps were employed in the test. In the analysis section of this study, it was concluded that the distortion resulting from photographic reproduction had little effect on response variation. However, it has been concluded that this distortion must be considered as a significant factor in the choice of quantization level in choropleth mapping. If the higher density portions of the data set are being generalized by the reproduction method, then the continuous tone map becomes no less a "useful lie" than the highly generalized choropleth map. This factor is one of several which should be considered in the selection of a quantization level. The other factors which were recognized in this study will be discussed in conclusions about the data and data analysis.

Original Data

The third major problem which was discovered in this project concerns the original data used to make the test maps. In selecting the original data, several trial maps were made, and evaluated according to the criteria defined in Chapter 3. These criteria were developed to evaluate the visual qualities of the original distribution, in an attempt to focus upon the map viewer, rather than upon the statistical qualities of the mapped data. It was not until the analysis stages that the measures for evaluating accuracy and complexity of the maps were selected and developed, and the effects which the nature of the original statistical distribution were to have on these measures were revealed. The linear distribution of the data was relatively undisturbed by the optimum generalization process, resulting in narrow value ranges of high accuracy and low complexity in the test maps. Thus, after the testing had been completed, it was realized that the nature of the statistical distribution would have more effect on the visual maps than was originally anticipated. However, while the use of a distribution which was markedly affected by the generalization process might have provided a broader statistical range in the test maps, it might have resulted in visual variations that were difficult, or impossible, to control. The qualities of the test maps appeared to affect the patterns of viewer response quite strongly. Although the range in response was low, an understanding of the original visual

and statistical qualities of the test maps allowed the isolation of possible sources and general patterns of response variation.

The problems created by the difficulties in controlling both the visual and statistical patterns in the data for mapping have prompted several conclusions in this study. First, the importance of isolating both the statistical and areal patterns of test maps in a project of this nature was recognized. Since both variables produce effects upon the patterns of viewer response to test maps, an understanding of these variables is important in the evaluation of viewer response. The second conclusion reached in observing the data problem is that a balance of the statistical and areal patterns of data when making a set of maps for a perception test is difficult to achieve. In attempting to control specialized qualities of the test maps, other qualities are lost. The final conclusion resulting from the data problem concerns both the area of this study, and the area of selection of a quantization level in constructing choropleth maps. It has become apparent through the evaluation of data patterns in this study that the statistical and visual patterns of a data distribution are not necessarily related in a predictable manner. For example, while two unquantized maps may possess similar visual distributions, the generalization of each statistical distribution may alter the areal patterns in very different ways. Essentially, the relationships between statistical and areal distribution patterns will vary with each set of original data, and at each quantization level. Thus, it must be concluded that an optimum quantization level suggested in this particular study could apply only to the data mapped in this study. More importantly, it has been concluded that selection of an optimum quantization level will depend strongly upon a clear understanding of the effects that the statistical characteristics of the data distribution will have on the visual patterns present in the mapped form.

Analysis

The final problem that prompted and influenced conclusions was the difficulty encountered in analysis of the two dimensional response maps collected in the study. After several unsuccessful attempts to apply standard statistical tests to the data, it was concluded that the task of data analysis should be carried out using tests and measures designed for the specialized nature of the data. The inapplicability of predictive tests in the data analysis, combined with conclusions about the potential variation between

statistical and visual distribution of test maps, persuade the author that specific conclusions about viewer response to the test maps cannot be used to make predictive statements about a general optimum quantization level in choropleth mapping. However, knowledge of the original mapped patterns, combined with observation of response patterns, may provide information from which conclusions about sources and patterns of response variation could be made. These conclusions could be applied in further tests of viewer perception, and in the practical selection of an optimum quantization level for a specialized data set and map audience.

Conclusion

Several major problems were encountered in this study. Although resolution of these problems required several changes to both the thought and practice of this project, their influence has inspired the most educational challenges and valuable conclusions in this thesis.

C. The Average Map Reader

Results of studies conducted by Jenks(1974) and Muller(1979) have provided evidence to support the existence of an "average" map reader. The theory of the average map has been very useful in the study of viewer perceptions in mapping, permitting the use of sampling methods and statistical analysis in perception research. The sampling in this study was undertaken on the assumption that an average, or representative, map could be compiled from a series of response maps. The normal distribution of deviations from the median response about the mean deviation level and the normal distribution of individual response accuracies about the mean accuracy for each subsample provide support for the theory of the average map reader. Statistical testing was not possible with the reponse arrays. However, the patterns of response between subsamples gain considerable impact from the fact that they are derived from a statistically representative composite response of the thirty individual responses for each subsample. The results of this study permit the conclusion that an average map reader does exist, and that the theory of the average map is a useful tool for the analysis and verification of viewer perception in mapping studies.

D. A Threshold Quantization Level ?

Introduction

The specific hypothesis upon which the procedure of this thesis was based is defined as the proposal that a threshold quantization level which optimizes the tradeoff between accuracy and cartographic communication exists for choropleth maps. Conclusions about problems with the sample size, uniqueness of choroplethic patterns, and inapplicability of statistical tests to the data arrays have resulted in an alteration of the original hypothesis. The altered hypothesis may be stated as follows: A unique threshold level of generalization which optimizes the tradeoff between accuracy and cartographic communication exists for each combination of original data set and proposed map audience in choropleth mapping. The observed patterns of response in the results of this study suggest an optimum generalization level for this data set, while the overall observations in this thesis provide an excellent set of guidelines for the selection of an optimum generalization for any data set in choropleth mapping.

Optimum Generalization Level for the Data in this Project

It appears from the results of this study that an optimum generalization of this data set is located at the fifty class generalization level. A levelling of response accuracy and consistency, after sharper trends to this stage from the more generalized test maps, commences consistently at this stage. In the cases of patterns in misranked units and mean response deviations, there is actually a declining trend at the fifty class level which breaks a general upward pattern. It is also at the 50 class stage that a levelling of the complexity and accuracy occurs, again after a sharp trend from the more generalized maps up to this stage. This pattern indicates that the addition of information beyond the fifty class generalization level would not enhance the map in any way. Considering, also, the distortion problems inherent in the photographic reproduction of choropleth maps, this observation gains considerable significance.

Conclusion

The fifty class generalization level represents an optimum tradeoff between accuracy (based on test maps) and cartographic communication (based on patterns of viewer response), which also encompasses the practical problems of choropleth map reproduction. The confidence with which a threshold generalization level was selected

for this data has promoted the conclusion that the means by which a threshold generalization level was selected will provide guidelines for the selection of an optimum level for other data sets.

E. Suggested Guidelines for the Selection of An Optimum Generalization

It is hypothesized that an optimum generalization level exists for all combinations of original data and potential map audience. The observations, results, problems, and conclusions have been used to develop a set of guidelines for the selection of this threshold level. The guidelines are outlined briefly as follows:

1. The cartographer must begin with specific knowledge of the purpose and proposed audience for the map, and establish whether a choropleth map which optimizes the tradeoff between accuracy and cartographic communication will be the best choice for his situation.
2. Knowledge of both the statistical and visual patterns of the data distribution at both original and generalized levels must be obtained. The general qualities and trends of these patterns could be discerned through the examination of a relatively small generalization range of the data.
3. The next step involves the isolation of factors which might influence cartographic communication. Since the goal of an optimum generalization level is the balance between accuracy and cartographic communication, accuracy levels will be a key factor to be observed. Complexity may be viewed as a means by which the level of information that will be transferred as accuracy increases may be predicted. If accuracy and complexity increase along similar paths, as observed here, then selection of a threshold generalization level will be conducted with the greatest degree of ease. Variations in the relationship between accuracy and complexity at different generalization levels may render the choice more difficult. Another observed factor influencing cartographic communication is the detrimental effect of photographic reproduction on the representation of grey tones on choropleth maps. Even though the finest quality of map production was selected, some of the quality was lost in reproduction of original maps. Map accuracy becomes dependent upon reproduction quality as well as information level; if distortion will lower the

accuracy level of a continuous tone map, thereby misleading the map viewer, then the cartographer should select a generalization level at which the accuracy of the information is displayed honestly by the reproduced map which the reader sees.

4. The final suggested guideline is encouragement to use the potential of computer technology as both an analytical and descriptive tool in evaluating the qualities of a series of choropleth maps before selection of a generalization method and generalization level. All the suggested factors outlined here can be evaluated and visually presented using both original and packaged computer programs. The packaged and previously used programs permit ease of analysis and description, while the construction of original programs permits indefinite variety in the focus of analysis. The emphasis here is put upon the use of the computer in the preliminary stages of selecting an optimum quantization level, as a potentially powerful tool in map design.

F. The Quantization Controversy

The original problem which inspired an interest in this area of thematic cartography was the increasingly heated debate over generalization levels in choropleth mapping, most recently manifested in the debate over quantized versus unquantized maps. In the conclusions to this project the issue is revisited, and discussed according to the results and experiences discovered since the controversy was first observed.

The results of data analysis corroborate the results of Muller(1978) and Peterson(1979), by exhibiting that people can obtain meaningful levels of information from choropleth maps at a broader range of generalization than previously anticipated. The general findings and observations of the thesis indicate that many complex factors which will influence the final mapped product must be considered in selection of a quantization level. It has been hypothesized that a unique optimum quantization level will exist for all data sets.

Thus, the final conclusion in this thesis concerning the quantization controversy in choropleth mapping is that its continued existence is unnecessary. The potential exists for the construction of choropleth maps at any quantization level, using any generalization technique. It must be accepted that if the purpose of the map, the potential map audience,

and patterns of original and generalized distributions of the data are understood to the fullest possible level, then the right choices in choropleth map construction will be made for that particular situation.

G. Cartographic Communication in Choropleth Mapping

The current emphasis on cartographic communication in thematic cartography, and the accepted definition of cartographic communication as the transmission of meaningful information via the, map formed the framework for this research, based on the assumption that this framework was applicable to the problems and purposes of choropleth mapping. The findings of this study have reinforced the original belief that an emphasis on cartographic communication would result in better use of the choropleth medium in thematic cartography.

The findings of data analysis have provided evidence that meaningful information is transmitted via the choropleth map with a high number of classes. The factors of accuracy and complexity were graphed in patterns which exhibited the reverse of patterns in response accuracy and consistency, indicating a correlation between accuracy and cartographic communication. In this case, the patterns of quantization level and accuracy did not follow similar patterns, as seen in a comparison of Figures 5 and 14. For this data set, as generalization level changed gradually from three classes to twenty, the accuracy level and complexity level jumped sharply. As the generalization level jumped sharply to the unquantized map, accuracy levelled off. The patterns of complexity variation were even more interesting. Complexity was measured according to contrast levels over the test maps. The highest level of contrast at the seven class level suggested the existence of a threshold generalization level, at which data were classified to the extent that the smoothness and patterns of original distribution were lost, without the benefit of obvious distinctions found on a more generalized map. The lack of unity between patterns of accuracy, complexity, and generalization levels exhibits the necessity of careful definition and study of the factors affecting cartographic communication when designing a choropleth map. The existence of relationships is seen, but the details of these relationships will likely change with each data set mapped.

Owing to the time and cost involved in conducting a perception study in choropleth mapping, it appears doubtful that such an extensive study would be undertaken as a preliminary phase of choropleth map design. However, the general findings of this study show the potential for detailed understanding of the patterns in data and generalizations as well as the characteristics of choropleth maps affecting cartographic communication, before the final generalization is selected. Thus, it is concluded that a direct concern with the viewer's ability to extract meaningful information from the choropleth map can be dealt with most successfully in an indirect way; through the acceptance of the suggested guidelines of choropleth map design, which focus upon those features of the choropleth map which may affect cartographic communication.

H. Conclusions Reached About Cartographic Communication Theory

It was originally proposed that the findings of this research might provide insight into the development of cartographic theory in thematic cartography. The research has definitely shown that the accepted conventions in choropleth mapping lose validity in the current framework of technological change and cartographic communication in thematic cartography, leading to the conclusion that cartography cannot develop progressively based on the acceptance of cartographic conventions as theory. The opposite extreme of utilizing all the new potential for map design and production, without setting a framework, might also impede a general level of progress within the discipline.

The procedures used in this study are a small example of the new possibilities in production and research techniques. The results of analysis are an example of the complex array of inter-related features which must be considered in map design. The expanding proliferation of considerations in thematic cartography makes the development of a cartographic theory difficult. It appears that an acceptance of the paradigm of cartographic communication and an ongoing awareness of the changing factors in cartography affecting cartographic communication must suffice as the current framework for the development of thematic cartography. In all areas of thematic cartography, the cartographer should not hesitate to break from cartographic convention in an attempt to utilize the potential of new techniques of production and analysis. It is

concluded that developments and research in thematic cartography can move in a progressive fashion, both practically and theoretically; acceptance of the necessity to focus on an understanding of the relationship between the paradigm of cartographic communication and rapid changes in thematic cartography may result in an evolutionary development of cartographic theory.

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Appendix A

TEST INSTRUCTIONS

Objective:

You are being asked to generalize the information displayed on this map into three distinct categories of high, medium, and low density. The darkest areas are those of highest density, and the lightest areas are those of lowest density.

Task:

Please use the supplied pen to subdivide the map into the three categories, and to label them accordingly. When you are finished, each part of the map should be contained in either a high, medium, or low density group.

Use the following abbreviations when labelling the density groups:

high - H
medium - M
low - L

If you have any questions, please ask them.

Return the pen and question sheet when you are finished.

7

10

20

50

100

183

Appendix B

A. Documentation for CROSS

1. Language: Pascal
2. Purpose: to calculate the number of deviations from median response by enumeration unit and respondent in one, two, and three dimensions from a response matrix containing a response value of 1,2, or 3 for each unit. (note: for this thesis, only the deviations in one dimension by unit were used in the final analysis)
3. Restrictions:
   a. maximum number of units is 200
   b. maximum number of respondents is 30
   c. response values must be 1, 2, or 3.
4. Input Requirements:
   a. Input file made up of 183 numbered units down vertical axis, by 30 respondents and one or two comparison maps(eg. median map) across horizontal axis. Matrix values match response of 1, 2, or 3 with each unit.(sample listing in Appendix D)
   b. Unit numbers in first three columns of file, followed by response values separated by one space.
5. Running the Program:
   a. Initial command; $RUN CROSS PAR=(leave a blank) SAMPLE=input file
   b. Prompts;
      1) first prompt gives the choice of calculating deviations from the first comparison map(in this case, median map) or the second comparison

map(optimum or other 3 class generalization). for first choice, type in 1; for second choice, type in 2.

2) second prompt gives the choice of output sorted according to unit number, or unit density value. for first choice, type in 1; for second choice, type in 0(zero).

6. Results:

a. results listed in specified output form

b. placed in temporary file -DIFFER

c. program prompts user to truncate and rename temporary file if it is to be saved

d. sample listing in Appendix D

```
{ Calculate difference statistics by respondent and by mapzone in 1,2,3
    dimensions for 3 classification levels }
[ J.L.Honsaker  --- 81-7-22 }
$ST-$
PROGRAM CROSS( SUMMARY, SAMPLE, OUTPUT );
CONST Samples = 30;            Units = 200;  Lastline = 183;
TYPE  HML   =  1..3;
      Coltype = ( Median, Jenks );
      Respons = 1..Samples;
      Areas   = 1..Units;
      People = ARRAY [Respons] of INT;
      Places = ARRAY [Areas] of INT.
VAR   Rhigher, Rlower, Class  : People;
      Uhigher, Ulower  : Places;
      Different         : ARRAY [HML] of People;
      Classcount        : ARRAY [HML] of INT;
      Ref  :  Coltype;
      ID, Model, Unitorder, Who, Diff, Lines  : INT;
      SAMPLE, SUMMARY  : TEXT;
      Disagree, Dist2, Dist3 :  REAL;
PROCEDURE  ACCUMULATE;
VAR   Class     : People;
      Deriv     : INT   ;
BEGIN
      READ (SAMPLE, ID );
         IF ID > Units then
                  Begin
                  WRITELN ('O  **** Incorrect Unit ID:',ID,
                              '  on line number:', Lines) ;
                  HALT;
                  End;
      FOR Who := 1 to Samples DO
               READ (SAMPLE,  Class (Who) );
      IF Model = 2 THEN READ ( SAMPLE, Deriv);  { skip first }
      READLN (SAMPLE,  Deriv );
      FOR Who := 1 to Samples DO
           Begin
               IF Class (Who) < Deriv  then
                 Begin
                   INCR ( Rlower (Who) );
                   INCR ( Ulower (ID)  );
                   INCR ( Different (Deriv, Who) );
                 End;
               IF Class (Who) > Deriv  then
                 Begin
                   INCR ( Rhigher (Who) );
                   INCR ( Uhigher (ID)  );
                   INCR ( Different (Deriv, Who) );
                 End;
           End;
         WRITE ( SUMMARY, ID :6 );
         Disagree  := Ulower (ID) + Uhigher (ID);
         Dist2 := SORT(SOR( Ulower (ID)) + SOR( Uhigher (ID)) ) ;
         WRITELN (SUMMARY, Disagree :10:2, Dist2 :10:2);
      INCR ( Classcount (Deriv) );
END;
PROCEDURE INITIALIZE;
BEGIN
      FOR Who := 1 to Samples DO
                  Begin
                  Rlower (Who) := 0;  Rhigher (Who) :=0;
                  Different (1,Who) := 0;
                  Different (2,Who) := 0;
                  Different (3,Who) := 0;
          End;
         FOR ID := 1 to Units DO
                  Begin
                  Ulower (ID) := 0;  Uhigher (ID) := 0;
                  End;
END;
{ Main program }
BEGIN
      RESET ( SAMPLE  );
      REWRITE ( SUMMARY, '-DIFFER ' );
      INITIALIZE;
      WRITELN (OUTPUT, '& Median (1) or Jenks (2) ?' ) ;
      READ (INPUT, Model );
      CASE Model of
            1: Ref:= Median;
            2: Ref:= Jenks ;
      End; { case }
      WRITELN (SUMMARY, '   Unit     Disagree   Dist.2  ',Ref ) ;
      WRITELN;
      FOR Lines := 1 to Lastline Do ACCUMULATE;
      WRITELN (SUMMARY, ' Response  Disagree   Dist.2    Dist.3');
      FOR Who := 1 to Samples DO
            Begin
                WRITE ( SUMMARY, Who :6 );
                Disagree  := Rlower (Who) + Rhigher (Who);
                Dist2 := SORT(SOR( Rlower (Who) ) + SOR( Rhigher (Who) ) );
                Dist3 := SOR ( Different (1,Who) ) + SOR ( Different (2,Who));
                Dist3 := SORT ( Dist3 + SOR ( Different (3,Who) ));
        WRITELN (SUMMARY, Disagree:10:2, Dist2:10:2, Dist3:10:2  );
            End;
{  now the statistics measured by area units }
      WRITELN (SUMMARY, '   Unit     Disagree   Dist.2   ');
      WRITELN (OUTPUT, '& Ordered Unit table (1) or not (0) ?' ) ;
      READ ( INPUT, Model );
      IF Model = 1 THEN Begin
      FOR ID := 1 to Units DO
            Begin
                WRITE ( SUMMARY, ID :6 );
                Disagree  := Ulower (ID) + Uhigher (ID);
                Dist2 := SORT(SOR( Ulower (ID)) + SOR( Uhigher (ID)) ) ;
                WRITELN (SUMMARY, Disagree :10:2, Dist2 :10:2);
             End;      { for }          End;  { if }
      WRITELN ('O  File "-DIFFER" has been created. TRUNC and RENAME to save it');
END.
```

B. Documentation for OAI

1. Language: Fortran

2. Purpose:

   a. to calculate the overview accuracy index for choropleth maps with up to 200 classes, based on class limit values of each class, as set out on a standard listing of the original units, unit values, and unit densities from which the generalization was constructed.

3. Restrictions:

   a. limit of 200 classes

   b. limit of 200 original units, densities, and areas

4. Input Requirements:

   a. an array of original unit numbers, density values, and unit areas; typed in this order; program will prompt for the Fortran format of this file

   b. class limit values, beginning with upper limit of lowest class and ending with lower limit of highest class; values separated by commas(free format)

5. Running the Program:

   a. Initial command; $RUN OAI 1=unit,density,area file 2=class limits file 3=output file

   b. Prompts;

      1) input number of units and number of classes; respond with the two values, respectively, separated by commas

      2) input format for units, densities, and areas

file; respond with Fortran format

6. Results:

   a. lists the following statistics in output file attached to unit 3

      1) overall deviations

      2) number of elements in each class and deviation for each class

      3) overview accuracy index for the map specified on unit 2

```
C--PROGRAM TO COMPUTE ACCURACY INDEX ON CHOROPLETH MAPS
C--ACCORDING TO JENKS OAI FORMULA
      DIMENSION D(200),A(200),RL(200),ASUM(200),AR(200),DD(200),FMT(6)
      COMMON M,N,D,A,RL,PSUM,N1
      LOGICAL*1 FREE(1)/'*'/
      WRITE(6,200)
 200  FORMAT(2X,'INPUT THE NUMBER OF COUNTIES AND CLASSES')
      READ(5,FREE)M,N
C--INPUT DENSITIES AND AREA OF UNITS
      WRITE(6,300)
 300  FORMAT(2X,'INPUT FORMAT FOR DENSITIES AND AREAS')
      READ(5,301)FMT
 301  FORMAT(6A4)
      READ(1,FMT)(D(I),A(I),I=1,M)
      N1=2*N-2
C--INPUT THE CLASS LIMITS,ARRANGED FROM LOWER TO HIGHER CLASS
      READ(2,FREE)(RL(I),I=1,N1)
C--CALCULATE THE MEAN
      SSUM=0.0
      TOTAL=0.0
      DO 500 I=1,M
      TOTAL=TOTAL+A(I)
 500  SSUM=SSUM+D(I)*A(I)
      AMEAN=SSUM/TOTAL
C--CALCULATE OVERALL WEIGHTED SQUARE DEVIATIONS
      GSUM=0.0
      DO 501 I=1,M
 501  GSUM=GSUM+((D(I)-AMEAN)*(D(I)-AMEAN)*A(I))
      WRITE(3,150)GSUM
 150  FORMAT(///,5X,'OVERALL DEVIATIONS =  ',F14.1,///)
C--CALCULATE O.A.I
      PSUM=0.0
      CALL OAI
      OAII=1.0-PSUM/GSUM
      WRITE(3,103)N,OAII
 103  FORMAT(///,2X,'OAI ACCURACY FOR THE ',I3,1X,'CLASS MAP =',F12.6)
 100  FORMAT(6X,F6.2,3X,F4.2)
      STOP
      END
C
C
C
C
      SUBROUTINE OAI
C--CALCULATE DEVIATIONS FROM THE MEAN IN EACH CLASS
      COMMON M,N,D,A,RL,PSUM,N1
      DIMENSION D(200),A(200),RL(200),ASUM(200),AR(200),DD(200)
      DO 20 K=1,N
      L=0
      AS=0.0
      S=0.0
      SUM=0.0
      IF(K.EQ.1) GO TO 30
      IF(K.EQ.N) GO TO 35
      IF(K.NE.2) GO TO 40
      LI=K
      LS=LI+1
      CALL SUBOAI(M,D,A,RL,LI,LS,SUM,L)
      ASUM(K)=SUM
      WRITE(3,100)K,L,ASUM(K)
```

```
 100  FORMAT(2X,'NUMBER OF ELEMENTS IN CLASS',I4,' IS',I4,' WITH DEVIATI
     1ON',F10.1,//)
      GO TO 20
  40  LI=LS+1
      LS=LI+1
      CALL SUBOAI(M,D,A,RL,LI,LS,SUM,L)
      ASUM(K)=SUM
      WRITE(3,100)K,L,ASUM(K)
      GO TO 20
  30  CONTINUE
      DO 50 I=1,M
      IF(D(I).GT.RL(1)) GO TO 50
       L=L+1
      AR(L)=A(I)
      DD(L)=D(I)
      AS=AS+AR(L)
      S=S+DD(L)*AR(L)
  50  CONTINUE
      AMEAN=S/AS
      DO 55 I=1,L
      SUM=SUM+((DD(I)-AMEAN)**2)*AR(I)
  55  CONTINUE
      ASUM(K)=SUM
      WRITE(3,100)K,L,ASUM(K)
      GO TO 20
  35  CONTINUE
      DO 60 I=1,M
      IF(D(I).LT.RL(N1)) GO TO 60
      L=L+1
      AR(L)=A(I)
      DD(L)=D(I)
       AS=AS+AR(L)
      S=S+DD(L)*AR(L)
  60  CONTINUE
      AMEAN=S/AS
      DO 65 I=1,L
      SUM=SUM+((DD(I)-AMEAN)**2)*AR(I)
  65  CONTINUE
      ASUM(K)=SUM
      WRITE(3,100)K,L,ASUM(K)
  20  CONTINUE
C--CALCULATE THE SUM OF THE DEVIATIONS FOR ALL CLASSES
      DO 80 K=1,N
  80  PSUM=PSUM+ASUM(K)
      RETURN
      END
C
C
C
      SUBROUTINE SUBOAI(M,D,A,RL,LI,LS,SUM,L)
      DIMENSION A(200),D(200),RL(200),AR(200),DD(200)
      L=0
      AS=0.0
      S=0.0
      DO 10 I=1,M
      IF(D(I).GE.RL(LI).AND.D(I).LE.RL(LS)) GO TO 20
      GO TO 10
  20  L=L+1
      AR(L)=A(I)
      DD(L)=D(I)
```

```
      AS=AS+AR(L)
      S=S+DD(L)*AR(L)
   10 CONTINUE
      AMEAN=S/AS
      SUM=0.0
      DO 30 I=1,L
      SUM=SUM+((DD(I)-AMEAN)**2)*AR(I)
   30 CONTINUE
      RETURN
      END
```

C. Documentation for APL function NEWTRI

1. Language: APL

2. Purpose:

   a. to establish adjacency triads(three units on a map, each of which is adjacent to the other two) of units on a map, using a logical matrix of 183x183 with a value of 1 indicating adjacent pairs of units, and a value of 0 indicating non-adjacent pairs of units

3. Restrictions:

   a. number of triads limited to 144; to alter, change value in line 3

   b. size of matrix is limited to a maximum of 183x183; to alter, change values in lines 6, 11, 21

4. Input Requirements:

   a. logical matrix of 183x183 stored in APL variable(ADJ)

   b. a counter for triads, also an APL variable(TRIADS)

   c. a storage space for triad listing, an APL variable(TRI)

5. Running Function: type in function name; NEWTRI

6. Results:

   a. number of triads is automatically printed

   b. list of triads is located in variable TRI

```
[0]         NEWTRI
[1]         PAIR←0
[2]         TRIADS←0
[3]         TRI←(144,3)ρ0
[4]         A←0
[5]     SET:A←A+1
[6]         →(A>183)/QUIT
[7]         I←1
[8]         J←1
[9]    LOOP:J←J+1
[10]    NEW:→((ADJ[A;I])=0)/RESET
[11]        →(J=184)/RESET
[12]        →((ADJ[A;I])×(ADJ[A;J])=0)/LOOP
[13]        →(ADJ[I;J]=0)/LOOP
[14]        PAIR←ADJ[A;I]×ADJ[A;J]
[15]        PAIR←PAIR×(I>A)
[16]        TRIADS←TRIADS+PAIR
[17]        →(PAIR=0)/LOOP
[18]        TRI[TRIADS;]←UNITS[A],UNITS[I],UNITS[J]
[19]        →LOOP
[20]  RESET:I←I+1
[21]        →(I=183)/SET
[22]        J←I+1
[23]        →NEW
[24]   QUIT:'NUMBER OF TRIADS'
[25]        TRIADS
```

D. Documentation for APL function CONTRAST

1. Language: APL
2. Purpose:
   a. to calculate the complexity/contrast index for a map, based on the formula listed in Chapter 4, and using the triads calculated by function NEWTRI as input
3. Restrictions:
   a. limited to a maximum of 144 triads; to alter, change value in lines 3 and 7
   b. maximum contrast value set for these maps; to alter, change value in line 18
4. Input Requirements:
   a. variable containing list of triads(TRI)
   b. variable containing list of unit density values, located on line 9(in this case, named C7)
5. Running Function: type in function name; CONTRAST
6. Results:
   a. complexity/contrast index for values specified in line 9 is automatically typed

```
 [0]           CONTRAST
 [1]           INDEX←0
 [2]           ABS←3⍴0
 [3]           VALS←(144,3)⍴0
 [4]           L←0
 [5]           M←0
 [6]           N←0
 [7]           SUMS←144⍴0
 [8]           A←1
 [9]           VALS←C7[TRI]
[10]      LOOP:L←VALS[A;1]
[11]           M←VALS[A;2]
[12]           N←VALS[A;3]
[13]           ABS←|(L-M),|(M-N),|(N-L)
[14]           SUMS[A]←+/ABS
[15]           A←A+1
[16]           →(A>144)/COMPUTE
[17]           →LOOP
[18]   COMPUTE:INDEX←(+/SUMS)÷73440
[19]           'COMPLEXITY CONTRAST INDEX'
[20]           INDEX
```

Appendix C

Table V.1 Complexity/Contrast Index for Optimum Maps

# Classes	Complexity/Contrast Index
3	.247
5	.290
7	.313
10	.294
15	.302
20	.298
50	.303
75	.303
100	.303
183	.303

Table V.2 Overview Accuracy Index for Test Maps

# Classes	Overview Accuracy Index
3	.8822
7	.9777
10	.9886
20	.9983
50	.9998
100	.9999
183	1.0000

Table V.3 Overview Accuracy Index for all Responses

	7 cl	10 cl	20 cl	50 cl	100 cl	183 cl
	.651	.357	.230	.511	.577	.292
	.512	.516	.503	.498	.649	.283
	.689	.442	.650	.645	.454	.157
	.084	.309	.563	.440	.436	.153
	.267	.510	.097	.376	.654	.398
	.525	.453	.796	.529	.503	.425
	.198	.149	.414	.689	.630	.419
	.605	.122	.534	.592	.715	.414
	.000	.247	.299	.283	.443	.583
	.698	.385	.162	.095	.423	.556
	.673	.234	.686	.000	.444	.136
	.850	.278	.000	.464	.330	.152
	.751	.641	.196	.265	.694	.476
	.756	.593	.000	.579	.480	.528
	.756	.403	.148	.411	.425	.552
	.210	.745	.400	.469	.728	.520
	.557	.546	.311	.484	.000	.344
	.453	.000	.186	.325	.699	.501
	.286	.669	.348	.334	.684	.792
	.494	.582	.000	.400	.637	.268
	.749	.468	.023	.538	.230	.532
	.591	.662	.438	.635	.169	.786
	.000	.096	.237	.411	.219	.636
	.235	.282	.561	.582	.669	.195
	.297	.471	.338	.671	.257	.409
	.422	.471	.478	.297	.303	.561
	.126	.548	.000	.420	.296	.776
	.712	.303	.525	.538	.422	.423
	.078	.000	.440	.674	.325	.470
	.247	.716	.709	.590	.466	.607
MEAN	.454	.411	.342	.458	.465	.435
S.D.	.250	.189	.228	.165	.187	.195

Table V.4 Overview Accuracy Index for Median Responses

# Classes	Overview Accuracy Index
7	.7632
10	.7392
20	.7298
50	.7102
100	.7082
183	.6765

Table V.5 Number of Misranked Units in Response Maps

# Classes	High	Medium	Low	Total
7	6	16	0	22
10	12	27	6	45
20	7	21	22	50
50	14	21	10	45
100	12	32	7	47
183	13	26	26	65

Table V.6 Deviations from Median Response

# Classes	Mean	Standard Deviation
7	5.60	4.84
10	6.54	5.39
20	6.60	5.54
50	5.45	5.29
100	6.80	5.59
183	7.10	4.94

Appendix D

ORIGINAL DATA VALUES

Six of these value sets were used to construct the test maps used in this project. The other value sets were used in deriving a pattern of map complexity, and were also in reserve for further testing, if deemed necessary. The ID numbers correspond to those of the enumeration units for Toronto, and the data are ranked from highest to lowest value according to unit density. The values range from 255 to 0, since this is the range accepted by the facsimile method for map construction. The original unscaled values ranged from 100 to 0.

ID	183 CL	150 CL	100 CL	75 CL	50 CL
13	255.00	255.00	254.76	254.76	254.33
194	254.52	254.52	254.76	254.76	254.33
65	253.46	253.46	253.46	252.70	254.33
123	252.40	252.31	252.31	252.70	251.52
62	252.23	252.31	252.31	252.70	251.52
136	250.97	250.97	250.73	250.73	251.52
63	250.50	250.60	250.73	250.73	251.52
31	248.39	248.39	248.39	248.39	247.24
64	246.09	246.09	246.09	246.09	247.24
66	238.00	238.00	238.00	238.00	236.73
34	235.46	235.46	235.46	235.46	236.73
7	232.86	232.86	232.68	232.07	232.07
3	232.50	232.50	232.68	232.07	232.07
33	231.47	231.47	231.47	232.07	232.07
4	231.47	231.47	231.47	232.07	232.07
102	228.30	228.30	228.30	228.30	228.30
124	225.80	225.80	225.46	225.46	224.43
14	225.12	225.12	225.46	225.46	224.43
88	224.40	224.40	223.74	223.74	224.43
167	223.71	223.71	223.74	223.74	224.43
15	223.12	223.12	223.74	223.74	224.43
12	220.00	220.00	220.00	220.00	220.00
128	217.06	217.06	216.66	216.66	216.05
174	216.27	216.27	216.66	216.66	216.05
91	214.82	214.82	214.82	214.12	216.05
5	214.01	214.01	213.76	214.12	212.89
190	213.52	213.52	213.76	214.12	212.89
175	211.15	211.16	211.15	211.15	212.89
60	208.41	208.41	208.41	207.54	207.54
192	207.19	207.11	207.11	207.54	207.54
133	207.03	207.11	207.11	207.54	207.54
172	199.47	199.47	199.47	198.60	197.94
119	198.59	198.59	198.16	198.60	197.94
122	197.74	197.74	198.16	198.60	197.94
89	195.97	195.97	195.97	195.97	197.94
32	193.45	193.45	193.45	192.46	192.46
59	191.48	191.48	191.48	192.46	192.46
92	186.75	186.75	186.26	186.26	185.56
185	185.77	185.77	186.26	186.26	185.56
35	184.16	184.16	184.16	184.16	185.56
169	180.35	180.35	180.35	180.35	180.35
90	174.83	174.83	174.83	174.09	173.35
121	173.79	173.71	173.71	174.09	173.35
37	173.64	173.71	173.71	174.09	173.35
57	171.16	171.16	171.16	171.16	173.35
87	168.92	168.92	168.92	168.92	167.43
70	167.04	167.04	166.69	166.69	167.43
180	166.34	166.34	166.69	166.69	167.43
56	164.65	164.65	163.96	163.58	163.58
58	163.80	163.80	163.96	163.58	163.58
54	163.42	163.42	163.96	163.58	163.58
36	162.44	162.44	162.44	163.58	163.58
39	158.38	158.38	158.01	158.01	158.01
118	158.01	158.01	158.01	158.01	158.01
21	157.64	157.64	158.01	158.01	158.01
51	153.73	153.73	153.19	153.19	152.76
189	153.13	153.13	153.19	153.19	152.76
47	153.00	152.95	153.19	153.19	152.76
135	152.90	152.95	153.19	153.19	152.76
42	151.04	151.04	151.04	151.04	152.76
131	149.20	149.20	148.50	148.50	148.50
49	148.42	148.42	148.50	148.50	148.50
85	147.89	147.89	148.50	148.50	148.50
156	143.62	143.62	143.12	143.12	143.12
29	142.95	142.88	143.12	143.12	143.12
137	142.80	142.88	143.12	143.12	143.12
30	139.70	139.70	139.70	139.70	139.70
80	136.12	136.12	136.12	134.97	134.97
61	135.00	134.85	134.59	134.97	134.97
25	134.71	134.85	134.59	134.97	134.97
10	134.06	134.06	134.59	134.97	134.97
41	133.09	133.09	132.69	132.38	131.76
114	132.60	132.49	132.69	132.38	131.76
132	132.39	132.49	132.69	132.38	131.76
55	131.44	131.44	131.44	132.38	131.76
11	129.28	129.29	129.29	129.29	131.76
165	126.11	125.96	125.96	125.61	124.83
50	125.80	125.96	125.96	125.61	124.83
48	124.91	124.91	124.91	125.61	124.83
113	122.50	122.50	122.50	122.50	124.83
168	120.27	120.27	119.67	119.67	118.80
69	119.46	119.37	119.67	119.67	118.80
129	119.27	119.37	119.67	119.67	118.80
155	118.47	118.46	118.46	118.15	118.80
154	118.46	118.46	118.46	118.15	118.80
101	118.44	118.46	118.46	118.15	118.80
45	117.25	117.25	117.25	118.15	118.80
68	113.88	113.74	113.52	113.52	112.77
166	113.61	113.74	113.52	113.52	112.77
99	113.07	113.07	113.52	113.52	112.77
43	111.74	111.65	111.65	111.65	112.77
151	111.56	111.65	111.65	111.65	112.77
184	105.15	105.07	104.72	104.72	104.25
116	105.00	105.07	104.72	104.72	104.25
52	104.62	104.62	104.72	104.72	104.25
94	104.11	104.11	104.72	104.72	104.25
157	102.39	102.39	102.39	101.40	104.25
120	100.99	100.90	100.90	101.40	99.63
24	100.82	100.90	100.90	101.40	99.63
158	99.47	99.47	99.47	99.00	99.63
117	99.47	99.47	99.47	99.00	99.63
139	98.81	98.81	98.53	99.00	99.63
78	98.25	98.25	98.53	99.00	99.63
100	93.82	93.82	93.30	93.30	92.87
38	93.36	93.36	93.30	93.30	92.87
95	92.73	92.73	93.30	93.30	92.87
93	91.57	91.57	91.57	91.57	92.87
163	85.25	85.25	84.85	84.11	83.60
164	84.45	84.45	84.85	84.11	83.60
173	84.05	84.05	83.62	84.11	83.60

22	83.63	83.63	83.62	84.11	83.60
97	83.19	83.19	83.62	84.11	83.60
186	82.64	82.64	82.33	81.74	83.60
53	82.12	82.12	82.33	81.74	83.60
108	81.26	81.14	81.14	81.74	80.12
127	81.03	81.14	81.14	81.74	80.12
193	79.84	79.84	79.84	79.10	80.12
83	78.36	78.36	78.36	79.10	80.12
171	76.02	76.02	75.69	75.69	75.03
138	75.56	75.53	75.69	75.69	75.03
8	75.50	75.53	75.69	75.69	75.03
67	74.80	74.79	74.54	74.54	75.03
162	74.78	74.79	74.54	74.54	75.03
40	74.38	74.29	74.54	74.54	75.03
196	74.19	74.29	74.54	74.54	75.03
26	72.86	72.77	72.58	72.30	72.30
23	72.68	72.77	72.58	72.30	72.30
130	72.18	72.19	72.58	72.30	72.30
142	71.48	71.48	71.48	72.30	72.30
20	70.00	69.96	69.78	69.78	69.78
191	69.91	69.96	69.78	69.78	69.78
46	69.43	69.43	69.78	69.78	69.78
161	67.16	67.16	67.16	67.16	65.86
104	65.66	65.66	65.66	64.81	65.86
159	64.75	64.75	64.38	64.81	65.86
71	64.02	64.02	64.38	64.81	62.81
72	63.12	63.12	62.50	62.50	62.81
105	62.68	62.68	62.50	62.50	62.81
77	62.18	62.10	62.50	62.50	62.81
81	62.03	62.10	62.50	62.50	62.81
74	60.47	60.47	60.16	60.16	58.78
126	59.86	59.86	60.16	60.16	58.78
103	58.79	58.69	58.58	58.22	58.78
176	58.60	58.69	58.58	58.22	58.78
181	58.35	58.16	58.58	58.22	58.78
79	57.98	58.16	57.68	58.22	58.78
160	57.38	57.38	57.68	58.22	58.78
195	55.70	55.70	55.49	55.49	54.74
134	55.28	55.28	55.49	55.49	54.74
75	53.23	53.23	52.68	52.68	54.74
44	52.14	52.14	52.68	52.68	51.04
96	51.00	50.82	50.82	50.82	51.04
112	50.86	50.82	50.82	50.82	51.04
170	50.78	50.82	50.82	50.82	51.04
82	50.76	50.82	50.82	50.82	51.04
115	50.68	50.82	50.82	50.82	51.04
27	49.24	49.24	48.85	48.85	48.37
76	48.47	48.47	48.85	48.85	48.37
140	47.40	47.40	47.40	46.63	48.37
84	45.87	45.87	45.87	46.63	44.49
16	43.10	43.10	43.10	43.10	44.49
152	39.52	39.31	39.31	39.31	38.83
141	39.23	39.31	39.31	39.31	38.83
125	39.17	39.31	39.31	39.31	38.83
110	37.40	37.40	37.40	36.46	38.83
150	35.52	35.52	35.52	36.46	33.41
86	33.82	33.82	33.82	32.71	33.41
18	32.64	32.64	32.16	32.71	33.41
111	31.68	31.68	32.16	32.71	33.41
73	27.53	27.53	27.27	27.27	26.32
183	27.02	27.02	27.27	27.27	26.32
98	26.06	26.06	26.06	25.36	26.32
28	24.67	24.67	24.67	25.36	26.32
107	22.50	22.50	22.13	22.13	22.13
182	21.75	21.75	22.13	22.13	22.13
106	20.28	20.22	20.06	19.55	19.55
109	20.17	20.22	20.06	19.55	19.55
19	19.73	19.73	20.06	19.55	19.55
153	19.13	19.13	18.77	19.55	19.55
187	18.42	18.42	18.77	19.55	19.55
188	14.17	14.17	14.17	14.17	14.17
17	10.00	10.00	10.00	10.00	8.44
1	6.89	6.89	6.89	6.89	8.44

ID	3 CL	5 CL	7 CL	10 CL	15 CL	20 CL
13	220.95	226.77	238.85	250.16	251.51	251.51
194	220.95	226.77	238.85	250.16	251.51	251.51
65	220.95	226.77	238.85	250.16	251.51	251.51
123	220.95	226.77	238.85	250.16	251.51	251.51
62	220.95	226.77	238.85	250.16	251.51	251.51
136	220.95	226.77	238.85	250.16	251.51	251.51
63	220.95	226.77	238.85	250.16	251.51	251.51
31	220.95	226.77	238.85	250.16	251.51	251.51
64	220.95	226.77	238.85	250.16	251.51	251.51
66	220.95	226.77	238.85	250.16	229.35	233.63
34	220.95	226.77	238.85	223.39	229.35	233.63
7	220.95	226.77	238.85	223.39	229.35	233.63
3	220.95	226.77	238.85	223.39	229.35	233.63
33	220.95	226.77	238.85	223.39	229.35	233.63
4	220.95	226.77	238.85	223.39	229.35	233.63
102	220.95	226.77	238.85	223.39	229.35	224.35
124	220.95	226.77	238.85	223.39	229.35	224.35
14	220.95	226.77	238.85	223.39	229.35	224.35
88	220.95	226.77	238.85	223.39	229.35	224.35
167	220.95	226.77	238.85	223.39	229.35	224.35
15	220.95	226.77	238.85	223.39	229.35	224.35
12	220.95	226.77	203.31	223.39	212.95	224.35
128	220.95	226.77	203.31	223.39	212.95	212.16
174	220.95	226.77	203.31	223.39	212.95	212.16
91	220.95	226.77	203.31	223.39	212.95	212.16
5	220.95	226.77	203.31	223.39	212.95	212.16
190	220.95	226.77	203.31	223.39	212.95	212.16
175	220.95	226.77	203.31	223.39	212.95	212.16
60	220.95	226.77	203.31	195.10	212.95	212.16
192	220.95	226.77	203.31	195.10	212.95	212.16
133	220.95	226.77	203.31	195.10	212.95	212.16
172	220.95	226.77	203.31	195.10	191.37	196.12
119	220.95	226.77	203.31	195.10	191.37	196.12
122	220.95	226.77	203.31	195.10	191.37	196.12
89	220.95	226.77	203.31	195.10	191.37	196.12
32	220.95	163.26	203.31	195.10	191.37	196.12
59	220.95	163.26	203.31	195.10	191.37	196.12
92	220.95	163.26	203.31	195.10	191.37	184.26
185	220.95	163.26	203.31	195.10	191.37	184.26
35	220.95	163.26	203.31	195.10	191.37	184.26
189	220.95	163.26	159.35	195.10	191.37	184.26
90	134.11	163.26	159.35	162.52	168.18	168.18
121	134.11	163.26	159.35	162.52	168.18	168.18
37	134.11	163.26	159.35	162.52	168.18	168.18
57	134.11	163.26	159.35	162.52	168.18	168.18
87	134.11	163.26	159.35	162.52	168.18	168.18

70	134.11	163.26	159.35	162.52	168.18	168.18
180	134.11	163.26	159.35	162.52	168.18	168.18
56	134.11	163.26	159.35	162.52	168.18	168.18
58	134.11	163.26	159.35	162.52	168.18	168.18
54	134.11	163.26	159.35	162.52	168.18	168.18
36	134.11	163.26	159.35	162.52	168.18	168.18
39	134.11	163.26	159.35	162.52	150.91	154.73
118	134.11	163.26	159.35	162.52	150.91	154.73
21	134.11	163.26	159.35	162.52	150.91	154.73
51	134.11	163.26	159.35	162.52	150.91	154.73
189	134.11	163.26	159.35	162.52	150.91	154.73
47	134.11	163.26	159.35	162.52	150.91	154.73
135	134.11	163.26	159.35	162.52	150.91	154.73
42	134.11	163.26	159.35	162.52	150.91	154.73
131	134.11	163.26	159.35	138.33	150.91	144.94
49	134.11	163.26	159.35	138.33	150.91	144.94
85	134.11	163.26	159.35	138.33	150.91	144.94
156	134.11	163.26	159.35	138.33	150.91	144.94
29	134.11	163.26	159.35	138.33	150.91	144.94
137	134.11	163.26	159.35	138.33	150.91	144.94
30	134.11	163.26	124.35	138.33	131.94	144.94
80	134.11	114.60	124.35	138.33	131.94	131.29
61	134.11	114.60	124.35	138.33	131.94	131.29
25	134.11	114.60	124.35	138.33	131.94	131.29
10	134.11	114.60	124.35	138.33	131.94	131.29
41	134.11	114.60	124.35	138.33	131.94	131.29
114	134.11	114.60	124.35	138.33	131.94	131.29
132	134.11	114.60	124.35	138.33	131.94	131.29
55	134.11	114.60	124.35	138.33	131.94	131.29
11	134.11	114.60	124.35	138.33	131.94	131.29
165	134.11	114.60	124.35	118.42	131.94	131.29
50	134.11	114.60	124.35	118.42	131.94	131.29
48	134.11	114.60	124.35	118.42	131.94	131.29
113	134.11	114.60	124.35	118.42	116.77	116.77
168	134.11	114.60	124.35	118.42	116.77	116.77
69	134.11	114.60	124.35	118.42	116.77	116.77
129	134.11	114.60	124.35	118.42	116.77	116.77
155	134.11	114.60	124.35	118.42	116.77	116.77
154	134.11	114.60	124.35	118.42	116.77	116.77
101	134.11	114.60	124.35	118.42	116.77	116.77
45	134.11	114.60	124.35	118.42	116.77	116.77
68	134.11	114.60	124.35	118.42	116.77	116.77
166	134.11	114.60	124.35	118.42	116.77	116.77
99	134.11	114.60	124.35	118.42	116.77	116.77
43	134.11	114.60	124.35	118.42	116.77	116.77
151	134.11	114.60	124.35	118.42	116.77	116.77
184	134.11	114.60	92.16	99.37	99.37	101.73
116	134.11	114.60	92.16	99.37	99.37	101.73
52	134.11	114.60	92.16	99.37	99.37	101.73
94	134.11	114.60	92.16	99.37	99.37	101.73
157	134.11	114.60	92.16	99.37	99.37	101.73
120	134.11	114.60	92.16	99.37	99.37	101.73
24	134.11	114.60	92.16	99.37	99.37	101.73
158	134.11	114.60	92.16	99.37	99.37	101.73
117	134.11	114.60	92.16	99.37	99.37	101.73
139	134.11	114.60	92.16	99.37	99.37	101.73
78	134.11	114.60	92.16	99.37	99.37	101.73
100	56.20	114.60	92.16	99.37	99.37	92.87
38	56.20	114.60	92.16	99.37	99.37	92.87
95	56.20	114.60	92.16	99.37	99.37	92.87
93	56.20	114.60	92.16	99.37	99.37	92.87
163	56.20	66.69	92.16	76.79	82.34	82.34
164	56.20	66.69	92.16	76.79	82.34	82.34
173	56.20	66.69	92.16	76.79	82.34	82.34
22	56.20	66.69	92.16	76.79	82.34	82.34
87	56.20	66.69	92.16	76.79	82.34	82.34
186	56.20	66.69	92.16	76.79	82.34	82.34
53	56.20	66.69	92.16	76.79	82.34	82.34
108	56.20	66.69	92.16	76.79	82.34	82.34
127	56.20	66.69	92.16	76.79	82.34	82.34
193	56.20	66.69	92.16	76.79	82.34	82.34
83	56.20	66.69	92.16	76.79	82.34	82.34
171	56.20	66.69	62.10	76.69	72.73	72.73
138	56.20	66.69	62.10	76.69	72.73	72.73
8	56.20	66.69	62.10	76.69	72.73	72.73
67	56.20	66.69	62.10	76.69	72.73	72.73
162	56.20	66.69	62.10	76.69	72.73	72.73
40	56.20	66.69	62.10	76.69	72.73	72.73
196	56.20	66.69	62.10	76.69	72.73	72.73
26	56.20	66.69	62.10	76.69	72.73	72.73
23	56.20	66.69	62.10	76.69	72.73	72.73
130	56.20	66.69	62.10	76.69	72.73	72.73
142	56.20	66.69	62.10	76.69	72.73	72.73
20	56.20	66.69	62.10	76.69	72.73	72.73
191	56.20	66.69	62.10	76.69	72.73	72.73
46	56.20	66.69	62.10	76.69	72.73	72.73
161	56.20	66.69	62.10	76.69	72.73	72.73
104	56.20	66.69	62.10	55.73	61.13	61.13
159	56.20	66.69	62.10	55.73	61.13	61.13
71	56.20	66.69	62.10	55.73	61.13	61.13
72	56.20	66.69	62.10	55.73	61.13	61.13
105	56.20	66.69	62.10	55.73	61.13	61.13
77	56.20	66.69	62.10	55.73	61.13	61.13
81	56.20	66.69	62.10	55.73	61.13	61.13
74	56.20	66.69	62.10	55.73	61.13	61.13
126	56.20	66.69	62.10	55.73	61.13	61.13
103	56.20	66.69	62.10	55.73	61.13	61.13
176	56.20	66.69	62.10	55.73	61.13	61.13
181	56.20	66.69	62.10	55.73	61.13	61.13
79	56.20	66.69	62.10	55.73	61.13	61.13
160	56.20	66.69	62.10	55.73	61.13	61.13
195	56.20	66.69	62.10	55.73	50.88	50.88
134	56.20	66.69	62.10	55.73	50.88	50.88
75	56.20	66.69	62.10	55.73	50.88	50.88
44	56.20	66.69	62.10	55.73	50.88	50.88
96	56.20	66.69	62.10	55.73	50.88	50.88
112	56.20	66.69	62.10	55.73	50.88	50.88
170	56.20	66.69	62.10	55.73	50.88	50.88
82	56.20	66.69	62.10	55.73	50.88	50.88
115	56.20	66.69	62.10	55.73	50.88	50.88
27	56.20	66.69	62.10	55.73	50.88	50.88
76	56.20	66.69	62.10	55.73	50.88	50.88
140	56.20	66.69	62.10	55.73	50.88	50.88
84	56.20	27.34	62.10	55.73	50.88	50.88
16	56.20	27.34	26.54	55.73	36.90	36.90
152	56.20	27.34	26.54	25.79	36.90	36.90
141	56.20	27.34	26.54	25.79	36.90	36.90
125	56.20	27.34	26.54	25.79	36.90	36.90
110	56.20	27.34	26.54	25.79	36.90	36.90
150	56.20	27.34	26.54	25.79	36.90	36.90

86	56.20	27.34	26.54	25.79	36.90	36.90
18	56.20	27.34	26.54	25.79	36.90	36.90
111	56.20	27.34	26.54	25.79	36.90	36.90
73	56.20	27.34	26.54	25.79	19.88	22.48
183	56.20	27.34	26.54	25.79	19.88	22.48
98	56.20	27.34	26.54	25.79	19.88	22.48
28	56.20	27.34	26.54	25.79	19.88	22.48
107	56.20	27.34	26.54	25.79	19.88	22.48
182	56.20	27.34	26.54	25.79	19.88	22.48
105	56.20	27.34	26.54	25.79	19.88	22.48
109	56.20	27.34	26.54	25.79	19.88	22.48
19	56.20	27.34	26.54	25.79	19.88	22.48
153	56.20	27.34	26.54	25.79	19.88	22.48
187	56.20	27.34	26.54	25.79	19.88	22.48
188	56.20	27.34	26.54	25.79	19.88	10.35
17	56.20	27.34	26.54	25.79	19.88	10.35
1	56.20	27.34	26.54	25.79	19.88	10.35

183 CLASS RESPONSE MATRIX

This matrix is a sample of the 30x183 matrices discussed in the text of this thesis. Across the horizontal axis are the responses, and down the vertical axis are the unit numbers from the original map. The 3,2, and 1 values are the individual responses to each unit on the response map by each respondent.

13	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
194	3	3	3	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
65	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
123	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3
62	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3
136	3	3	3	2	3	2	3	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2	3	3	3	3	3	3
63	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
31	3	3	3	3	3	3	3	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
64	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
66	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
34	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
7	2	3	3	3	3	2	3	3	3	3	3	3	3	3	3	3	3	3	3	2	3	3	3	2	3	2	3	3	3	3	3	3
3	2	3	3	3	3	2	3	2	3	3	3	3	3	3	3	3	3	3	3	2	3	3	3	2	3	2	3	3	3	3	3	3
33	3	3	3	3	3	3	3	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
4	2	3	3	3	3	2	3	3	3	3	3	3	3	3	3	3	3	3	3	2	3	3	3	2	3	2	3	3	3	3	3	3
102	3	3	3	3	3	2	3	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
124	3	3	3	3	3	2	3	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2	3	3	3	3	3	3
14	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
88	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
167	3	3	3	2	3	2	3	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2	3	3	3	3	3	3
15	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
12	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
128	3	2	2	2	3	2	3	2	3	3	2	2	3	3	2	3	3	2	2	3	3	3	3	2	3	2	3	3	3	3	3	3
174	3	3	3	3	3	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2	3	3	3	3	3	3	3	3
91	3	3	3	2	3	3	3	2	3	3	3	3	3	3	3	3	3	2	2	3	3	3	3	3	3	3	3	3	3	3	3	3
5	2	3	3	3	3	2	3	3	3	3	3	3	3	3	3	3	3	3	3	2	3	3	3	2	3	2	3	3	3	3	3	3
190	3	3	3	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2	3	3	3	3	3	3	3	3	3	3	3	3	3
175	3	3	3	3	3	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2	3	3	3	3	3	3	3	3
60	3	3	3	2	3	2	3	2	3	3	3	3	3	3	3	3	3	2	2	2	3	3	3	3	3	3	3	3	2	3	3	3
192	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
133	3	3	3	2	3	2	3	2	3	3	3	3	3	3	3	2	3	2	2	2	3	3	3	3	3	3	3	3	3	3	3	3
172	3	3	3	3	3	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
119	3	3	3	2	3	3	3	2	3	3	3	3	3	3	3	3	3	2	2	3	3	3	3	3	3	2	3	3	3	3	3	3
122	3	3	3	2	3	3	3	2	3	3	3	2	3	3	3	3	3	2	2	3	3	3	3	3	3	2	3	3	3	3	3	3
89	3	3	3	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2	3	3	3	3	3	3	3	3	3	3	3	3	3
32	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
59	3	3	3	2	3	2	3	2	3	3	3	3	3	3	3	3	2	2	2	2	3	3	3	3	3	3	3	3	2	3	3	3
92	3	3	3	2	3	2	3	2	3	3	3	3	3	3	3	3	3	2	2	2	3	3	3	3	3	3	3	3	3	3	3	3
185	3	3	3	2	3	2	3	3	3	3	2	3	3	3	3	3	3	2	2	3	3	3	3	3	3	3	3	3	3	3	3	3
35	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2	2	3	3	3	3	3	3	3	3	3	3	3	3	3
169	3	3	3	2	2	2	3	2	3	3	3	3	3	3	3	3	3	2	2	2	3	3	3	3	3	3	3	3	3	3	3	3
90	3	3	3	2	3	3	3	2	3	3	3	3	3	3	3	3	3	3	2	3	3	3	3	3	3	3	3	3	3	3	3	2
121	3	3	3	2	3	3	3	2	3	3	3	3	3	3	3	3	3	2	2	3	3	3	3	3	3	2	2	3	3	3	3	2
37	3	3	3	2	3	3	3	2	3	3	3	3	3	2	3	3	3	2	2	2	3	3	3	2	3	2	3	3	3	3	3	2
57	3	2	3	2	3	2	3	2	3	2	3	2	3	3	3	3	3	2	2	2	3	3	3	3	3	2	3	3	2	3	3	2
87	3	3	3	3	3	3	3	2	3	3	3	3	3	3	3	3	3	2	2	3	3	3	3	3	3	3	3	3	3	3	3	2
70	3	3	3	2	3	2	2	2	3	3	2	3	3	3	3	3	2	2	2	3	3	2	3	3	3	3	3	3	2	3	3	2
180	3	3	3	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2	3	3	3	3	3	3	3	3	3	3	3	3	2
56	3	2	3	2	3	2	3	2	3	2	3	2	3	3	3	3	3	2	2	2	3	3	3	3	3	2	3	3	2	3	3	2
58	3	2	3	2	3	2	3	2	3	2	3	2	3	3	3	3	2	2	2	2	3	3	3	3	3	2	3	3	2	3	3	2
54	3	2	3	2	3	2	3	2	2	2	3	2	3	3	3	3	3	2	2	2	3	3	3	3	3	2	3	3	2	3	3	2
36	3	3	3	3	3	3	3	2	3	3	3	3	3	3	3	2	3	2	2	2	3	2	3	2	3	2	3	3	3	3	3	2
39	3	3	3	2	3	3	3	2	3	3	3	2	3	2	3	2	2	2	2	2	3	2	3	2	3	2	3	3	2	3	3	2
118	3	3	3	2	3	2	3	2	3	3	3	3	3	3	3	3	3	2	2	2	3	3	3	3	3	2	3	3	2	3	3	2
21	3	2	3	2	3	2	3	2	3	3	3	2	3	3	2	3	2	2	2	2	3	2	3	2	2	2	3	3	2	3	3	2
51	2	3	2	3	3	2	3	2	3	3	3	3	3	3	2	3	3	2	2	2	3	2	3	3	3	2	3	3	2	3	3	2
189	3	3	3	2	3	3	3	3	3	3	3	3	3	3	3	3	3	2	2	3	3	3	3	3	3	3	3	3	3	3	3	2
47	2	3	2	3	3	3	2	3	3	3	3	3	3	2	2	3	3	2	2	2	3	3	3	2	3	2	3	3	2	3	3	2
135	3	2	2	2	2	2	3	2	2	3	2	2	3	3	3	3	2	2	2	3	3	2	3	2	2	2	3	2	2	3	2	2
42	3	2	3	2	3	2	3	2	2	2	3	2	3	3	3	3	2	2	2	2	3	2	3	3	3	2	3	3	2	3	3	2
131	3	3	3	2	3	2	3	2	3	3	3	3	3	3	3	2	3	2	2	2	3	2	3	3	3	2	3	3	2	2	3	2
49	2	3	2	3	3	2	3	2	3	3	3	3	3	3	2	3	3	2	2	2	3	2	3	3	3	2	3	3	2	3	3	2
85	3	3	3	2	3	2	3	3	3	3	2	3	3	3	3	3	2	3	2	3	3	2	3	3	3	3	3	3	3	3	3	2
156	3	2	3	2	2	2	3	2	3	2	2	2	3	3	2	2	2	2	2	2	3	2	3	2	2	2	3	2	2	3	2	2
29	3	2	3	2	3	2	3	2	3	2	2	2	3	3	2	3	3	2	2	2	3	2	3	2	3	3	2	3	2	3	3	2
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157	2	2	3	2	2	2	3	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	2	2	2	1	2	2	2	2	2	2
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158	2	2	3	2	2	2	3	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2	2	2	2	1	2	2	2	2	2	2
117	2	2	2	2	2	2	2	2	2	3	3	2	2	2	2	2	3	2	1	2	3	2	2	2	3	1	2	3	2	2	2	2
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95	2	2	2	2	2	2	2	1	2	2	2	1	2	2	2	2	2	2	2	1	3	2	2	2	3	1	2	2	1	2	2	1
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127	2	2	1	1	1	1	1	1	2	1	1	1	2	2	1	1	2	2	1	2	2	2	2	1	2	1	2	2	1	2	2	1
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46	2	1	2	2	2	1	[illegible]	1	2	2	2	1	2	2	1	1	2	2	1	1	2	2	1	2	1	1	1	2	1	1	2	1
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195	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1	2	1	1	1	1	1	1	2	1	1	1	1
134	1	2	2	1	1	1	1	1	1	1	1	2	2	1	1	1	2	2	1	1	2	2	1	1	2	1	1	2	1	1	1	1
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183	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
[illegible]	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1
28	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
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106	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
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19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
153	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
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17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
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This is a sample of the array of deviations from median response generated for each response subsample. The unit numbers are those of the enumeration units for Toronto, on the original maps. The disagree numbers are the absolute numbers of respondents who disagreed with the median response value for that unit. For example, everyone agreed with the median response for unit 13, while 4 people disagreed with the median response for unit 136.

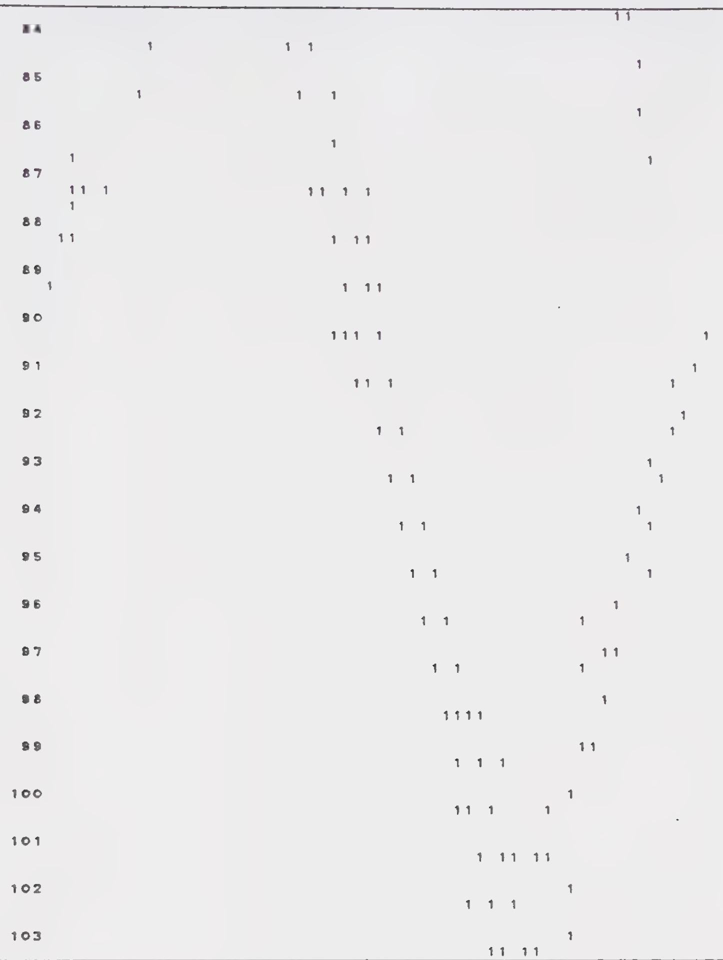
Unit	Disagree
13	0
194	1.00
65	0
123	1.00
62	1.00
136	4.00
63	0
31	1.00
64	0
66	0
34	0
7	5.00
3	6.00
33	1.00
4	5.00
102	2.00
124	3.00
14	1.00
88	0
167	4.00
15	0
12	0
128	12.00
174	2.00
91	4.00
5	5.00
190	2.00
175	2.00
60	7.00
192	0
133	7.00
172	1.00
119	5.00
122	6.00
89	2.00
32	0
59	8.00
92	6.00
185	5.00
35	2.00
169	7.00
90	3.00
121	6.00
37	8.00
57	11.00
87	3.00
70	10.00
180	2.00
56	11.00
58	12.00
54	12.00
36	8.00
39	13.00
118	8.00
21	16.00
51	11.00
189	3.00
47	11.00
135	12.00
42	14.00
131	11.00
49	11.00
85	6.00
156	10.00
29	15.00
137	12.00
30	15.00
80	10.00
61	14.00
25	8.00
10	12.00
41	14.00
114	11.00
132	14.00
55	14.00
11	13.00
165	9.00
50	14.00
48	15.00
113	12.00
168	15.00
69	9.00
129	8.00
155	8.00
154	9.00
101	10.00
45	15.00
68	8.00
166	14.00
99	10.00
43	12.00
151	8.00
184	13.00
116	6.00
52	8.00
94	6.00
157	4.00
120	8.00
24	9.00
158	4.00
117	8.00
139	9.00
78	9.00
100	6.00
38	12.00
95	7.00
93	7.00
163	15.00
164	13.00
173	9.00
22	8.00
97	9.00

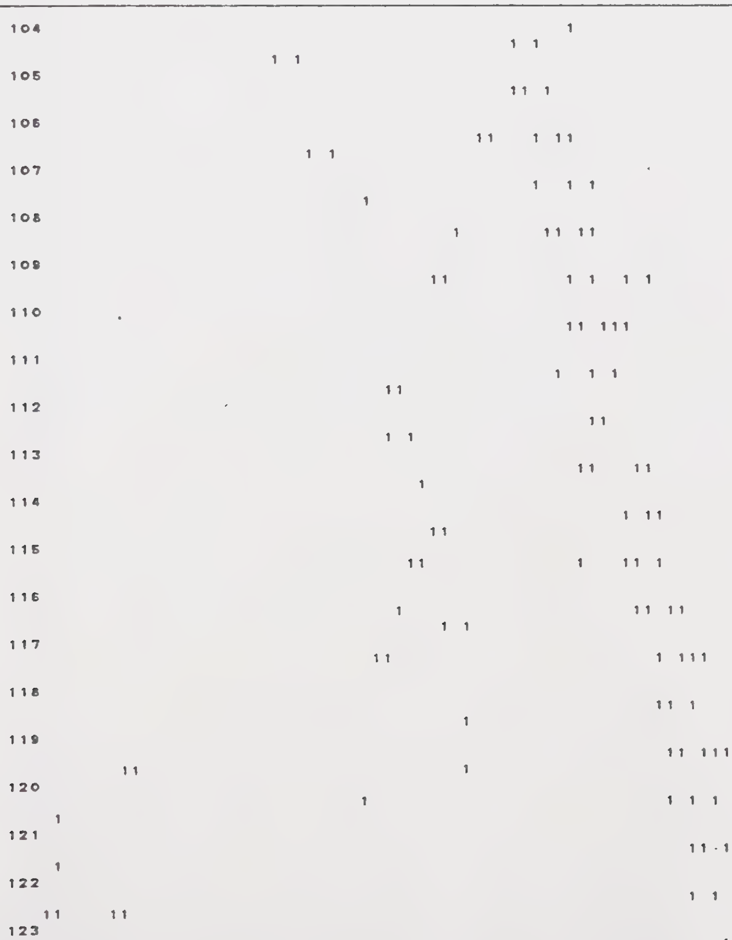
186	13.00
53	9.00
108	8.00
127	15.00
193	14.00
83	13.00
171	11.00
138	12.00
8	13.00
67	10.00
162	15.00
40	11.00
196	14.00
26	11.00
23	9.00
130	12.00
142	10.00
20	13.00
191	15.00
46	14.00
161	8.00
104	10.00
159	6.00
71	11.00
72	11.00
105	9.00
77	12.00
81	10.00
74	11.00
126	9.00
103	11.00
176	3.00
181	8.00
79	9.00
160	6.00
195	3.00
134	10.00
75	7.00
44	15.00
96	11.00
112	2.00
170	1.00
82	7.00
115	6.00
27	0
76	6.00
140	9.00
84	7.00
16	0
152	3.00
141	2.00
125	2.00
110	1.00
150	1.00
86	1.00
18	0
111	0
73	0
183	0
98	1.00
28	0
107	0
182	0
106	0
109	0
19	0
153	0
187	0
188	0
17	0
1	0

```
ADJACENCY MATRIX USED TO CALCULATE ADJACENCY TRIADS
   1             1111

   3  1 11                                       11

   4 1 11                                      111

   5  1 1                                    1  1

   7 111 1

   8 1  1 1 1

  10     1 1                             1  11

  11      1 1 1                      1  1

  12    1  1 11

  13        1 11 1

  14       111 1                   11

  15         11 1                  1

  16           1 11            1  1

  171        1  1

  181           1  1        11

  191             1       111

  201                1  111

  21                1 111

  22                 1 1
                     1

  23                 11 1
                    11


  24                11 1 1
                    1

  25                1   1 1
                   11

  26               11    1 1
                1

  27               1      1 1
               1

  28              11       1 1
               1

  29              1         1 11
           1

  30            1            1 1 1

  31                         11 1
          1

  32                           1 11
        1 1

  33            1             1 1 1

  34           1                11 1
     1

  35          1                   1 11
    1

  36       1                       1 1 1

  37                               11 1                      1

  38                                 1 1 1                 1

  39       1                        1 1 1

  40      1                            1 1 1

  41                                  1 1 1               1

  42                                     11   1         11

  43      1                             111 1
```

```
44   1  1                                1 11
45                                      1 1 1      11
46                                        11 1     1
47  11                                      1 1   1
48  1                                        1 1  1
49 11                                         1 111
50 1                                           1 1
                                          111
                      1
51                                             11 1
                                       1
52                                           11  1 1
                                       1
53                                         11     1 1
                                     11
54                                         1       1 1
                                    11
55                                      1           1 1
                                   1
56                                      1            1 11
                                  1
57                                                    1 1 1
                                 1
58                                     1              11 1
59                                  1                   1 11
60                                                     1 1 1
                                1
61                                 1                     11
  1                            1
62                               1                         1
   1                         1
63                              1
  1 1 1                     1
```

```
64
   1 11                     1
65
    1 11                   1
66                            1
   111 11                  1
67
     11 111
68                           11
      11 1
69                         1
       11 1111
70
       1 1 1            11
71
         11 1           1
72
         1 1 1 1       1
73                      11
         1  1 1
74                      1
             1 1 1
75
            1 1 1     1
76
               1 1  11
77                     1
              1 1 1
78                   111
                 1 11
79                  11
                  1 1
80
                1 11
                                                 11
81
                1     1
                                                  11
82
               1     1 1
                                                    1
83
            1         1 1
```





```
124      1  1
125     11                    1                                  111
126    1 1                   1                              1
127   1 1 11                      -                         1          1
128      1 1        1                                                  1
129      11 1       1
130   1    1 1     1                                            1
131        1 1  1                                           1   1
132         1  1                         11                    1
133            1                         11 1
134         111 1
135        11  1 1     1
136        1    1 1  1
137       1      1 1
138      1        1 11
139                1 1  1
140              1 11 11
141              1    1
142                   1  1
150                  1  1                     1 1
```

```
151                          11
152                         1 111
153                         11 1                 11
154                          11 111               1
155                          1 1  11              1
156                            1  1 1             1
157                            111 11
158                             1 1 1            1
159                              111 1           1
160                                 1 1         1 1
161                                  1 1       1     1
162                                   1 1      1      1
163                                    1 11   1
164                                    1  1           11
165                                     11 1  1        1
166                                       1  1        1 1
167          11                              1
168          11                          11 1
169           1                        1 1   1
170                                          1 1
```